IDENTIFICATION OF PRESTRESS FORCE IN PRESTRESSED CONCRETE BOX GIRDER BRIDGES USING VIBRATION BASED TECHNIQUES

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BSc (Hons)

Submitted in fulfilment of the requirements for the degree of
Doctor of Philosophy

School of Civil Engineering and Built Environment
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Queensland University of Technology
2017
Keywords

Box Girder Bridge, Inverse calculation, Prestressed concrete, Prestress force, Prestress identification, Vibration characteristics.
Abstract

Bridges form an important component of any transportation infrastructure system. Because of the importance of their role, bridges usually get more attention in designing and maintaining than other components in the transport system. They are usually designed for a higher service life of 100 years or more. Some of the current in-service bridges in Australia are more than 100 years old. These bridges are experiencing much higher traffic loads than their original design values. Therefore condition assessment of these bridges is vital to ensure their safe operation.

Among several types of bridges, prestressed concrete bridges are being widely used all over the world due to their superior overall performance. More than 60% of bridges among over 50000 bridges in the Australian road network are prestressed concrete bridges. The performance of these prestressed structures is governed by the effective prestress force which in fact reduces over time. A number of bridge failures all over the world due to faulty prestressing systems have drawn the attention of researchers towards assessing the effective prestress force in prestressed bridges using non-destructive methods. However, there have not been enough studies on prestressed concrete box girder bridges which are a major type of prestressed bridges. This study has therefore focused on filling this gap in knowledge by developing a novel method to quantify the effective prestress force of existing box girder bridges using their vibration responses.

Toward this aim, finite element study has been carried out to identify the effect of prestressing on the vibration characteristics of box girder bridges. Results show that the un-bonded prestressing force reduces the stiffness of the structure causing the natural frequency to reduce. Consequently, vibration responses also change with the magnitude of effective prestress force. A method was then developed to quantify this effect in an inverse calculation considering the plate-like behaviour of top slab of box girder bridges. A new approach to idealise the top slab of box girder bridges has been developed using boundary characteristic orthogonal polynomials. The scope of this research was limited to simply supported, single cell, straight, non-skew box girder bridges with uniform cross section. However, this method considers the
common feature of all types of box girder bridges so that the proposed method can be extended to all types of box girder bridges.

The proposed method has been tested with the experimental testing of a laboratory model of a prestressed concrete box girder bridge. Results show a good accuracy of the proposed method even with noisy measurements. It was evident that the novel method developed in this research can effectively identify the prestress force in box girder bridges using measured vibration responses due to external periodic excitation.
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List of Abbreviations

PSC  Prestressed concrete
E\textsubscript{conc}  Elastic modulus of concrete
E\textsubscript{steel}  Elastic modulus of steel
\rho\textsubscript{steel}  Mass Density of Steel
\rho\textsubscript{conc}  Mass Density of concrete
PF  Prestress Force
e  Eccentricity of effective prestress force
COP  Characteristic Orthogonal Polynomials
FEM  Finite Element
G\textsubscript{xy}  Shear modulus
\nu  Poisson’s ratio
\nu\textsubscript{xy}  Poisson’s ratio corresponding to strain in \textit{Y} direction for a load in \textit{X} direction
w\textsubscript{(x,y)}  Displacement of plate in \textit{Z} direction
E  Modulus of elasticity of plate material
\rho  Mass density of plate material
h  Plate thickness
A  Cross-sectional area
P\textsubscript{(x,y,t)}  Externally applied pressure
D  Bending stiffness of the plate
\omega\textsubscript{i}  \textit{i}th natural frequency of vibration
M  Mass of the beam (per unit length) or mass of plate (per unit area)
N  Axial force in beam (N) or in-plane load in plate (N/m)
N\textsubscript{x}  Axial force in \textit{X} direction
m, n  Mode number of plate
\omega\textsubscript{m,n}  \textit{m}, \textit{n}th natural frequency of a plate
a, b, L  Dimensions of the plate (As shown in relevant figures)
Y\textsubscript{(x)}  Mode shape function.
(EI)_b \quad \text{Bending stiffness of equivalent edge beam}

(GI)_b \quad \text{Rotational stiffness of the beam}

G_b \quad \text{Shear modulus of beam}

J \quad \text{Polar moment of inertia of the beam}

f \quad \text{Natural frequency of vibration}

\text{MSE} \quad \text{Mean squared error}

\bar{y}_i \quad \text{Response measured on box girder}

y_i \quad \text{Approximated response as a Euler Bernoulli beam}
Statement of Original Authorship

The work contained in this thesis has not been previously submitted to meet requirements for an award at this or any other higher education institution. To the best of my knowledge and belief, the thesis contains no material previously published or written by another person except where due reference is made.

Signature: QUT Verified Signature
Date: October 2017
Publications

Acknowledgements

My PhD journey of more than 3 and a half year was not an easy one. It would not be possible to write this thesis without the help of kind people around me to whom I’m greatly indebted.

I would first like to express my deepest appreciation to my principal supervisor Prof. Tommy Chan for his kind guidance throughout this journey. Professor Chan, it was a privilege to be a member of your research team. I appreciate the continuous help and support of my associate supervisors, Prof. David Thambiratnam and Dr Praveen Moragaspitiya. Assistance, experience and guidance of Dr Andy Nguyen the coordinator for my project was extremely helpful for the successful completion. Therefore my special thanks go to him for his support and kindness.

I gratefully acknowledge the financial support provided by the Queensland University of Technology. The experimental program of my research would not be so easy without the great support of Banyo laboratory staff. I’m grateful for all QUT Banyo pilot plant precinct staff for assisting and sharing their experience for making the lab testing a success. I also wish to thank QUT IT Help Desk, HPC unit, library staff and all other supporting staff for their support during my time at QUT.

I would like to express my gratitude to members of structural health monitoring research team and all my friends for sharing their knowledge and experience, and being with me and helping me at my hard times.

At last but not least I’m grateful to my father and all my family members, my wife and my son for being with me all times and for enormous encouragement and support.
Chapter 1: Introduction

Giving an introduction to the research, this chapter outlines the background (Section 1.1) and aim and objectives (Section 1.3) of the research. Section 1.4 describes the significance, scope of this research and context of the current study. Finally, Section 1.5 includes an outline of the remaining chapters of the thesis.

1.1 BACKGROUND

Prestressing is a process of applying an initial permanent stress onto an object or a structure before the application of usual in-service loads. Stresses due to service loads are then acting on top of this initial stress. Hence the final stress in structural elements can be controlled to remain within the desired range by carefully selecting the initial stress state.

Even though the term “prestressing” is relatively new, its basic concept has been used for more than 3500 years (Casson, 1971; Gasparini, 2006). Prestressing is being used in a number of applications in our day to day life to improve the load carrying capacity of objects. Shown in Figure 1-1 are two common examples. The first picture in Figure 1-1 is a wooden barrel used to store liquids. In these barrels staves are held by the means of pretensioned metal bands. They compress the staves providing the required strength to withstand the pressure from stored liquids. The second picture in Figure 1-1 is a bicycle wheel with radial spokes. Spokes play an important role in transferring load between the hub and the rim which also helps to keep the shape of the wheel and held the rim in position. They are initially tensioned between the rim and the hub. Compression that they experience due to applied loads in service stage is neutralised by the initial tension giving the ability to withstand the high compression which otherwise could have easily caused slender spokes to fail in buckling.
The same concept has been using effectively by civil engineers for many years to overcome the natural weakness of concrete in tension. It is a well-known fact that concrete is very strong in compression but weak in tension. Conventional method in dealing with tensile forces is to provide steel reinforcement to help resist tensile stresses developed in concrete. This method requires a considerably large section with a large amount of steel for long spanning structural elements such as bridge girders. Introduction of prestress to concrete can ensure that the final stresses due to applied loads remain within the capacity of concrete which also enables to use more slender members.

Prestressed concrete has gained its popularity over other conventional materials due to its better overall performance and has been used as an effective and economical bridge material for many decades. With first prestressed bridge built in the mid-1930s, prestressed concrete bridges became increasingly popular after the Second World War due to the great contribution by Eugène Freyssinet who is also known as the father of modern prestressing (Hewson, 2003b).

Prestress is usually applied to concrete by the means of external or internal tendons anchored to the concrete member. Internal tendons can be either bonded or unbonded. Depending on the time of cable tensioning, they can be further classified into two categories. It is called pre-tensioning if the tendons are tensioned before concreting and post tensioning if the tendons are tensioned after hardening concrete.
Main components of a typical modern prestressing system are common for both types and are shown in Figure 1-2.

![Figure 1-2 Components of concrete prestressing system (AMSYSCO, 2010)](image)

Both these methods apply an initial stress (essentially compressive) to the concrete which counteracts stresses that are developed due to self-weight and other working loads. These counteracting initial stresses are the main contributor that characterise the high load carrying capacity of prestressed elements which enable the prestressed concrete to effectively carry loads with much smaller sections and less material (both steel and concrete) compared to reinforced concrete. Giving an added advantage of self-healing, residual compression closes any crack formed due to overloading immediately after removing the load which also leads to crack free, durable structures.

1.2 RESEARCH PROBLEM

Effective prestress force in the tendons is the most important factor that determines the load carrying capacity of prestressed structures. However, prestress force in tendons reduces with time due to several reasons including creep and shrinkage of concrete and relaxation of steel. Any defects in prestressing system or damage to the strands such as corrosion can cause the prestress force to reduce significantly over the design considerations. Excessive reduction in effective prestress can lead to severe serviceability and safety problem. However, once stressed, there is still no effective method to determine the tension in the embedded prestressing tendons unless it is instrumented during construction.
Having identified the importance of prestress force as a governing factor for safety and well-functioning of prestressed bridges, a considerable amount of research emerged to predict the effective prestress force in in-service bridges. Some early methods of determining the residual stress in concrete such as stress release methods (Owens, 1993; Owens et al., 1994) utilize semi-destructive techniques which require making some damage to the structure. With recent advances in vibration-based methods in structural health monitoring, the trend has now turned towards using vibrational responses to determine the effective prestress which require no damage to the structure. However, almost all previous efforts in this manner were focused on prestressed beams (Bruggi et al., 2008; Caro et al., 2013; Changchun, 2003; Hamed & Frostig, 2006; Jang et al., 2011; Jang et al., 2013; Jang et al., 2010; Kim et al., 2003; Kim et al., 2004; Law & Lu, 2005; Law et al., 2008; Lu et al., 2008; Lu & Law, 2006; Osborn et al., 2012; Velez et al., 2010; Wang et al., 2008; Wu et al., 2008; Xu & Sun, 2011). No recorded effort on a successful method was found to evaluate the effective prestress level of box girder bridges which are another important form of prestressed bridge structures.

1.3 AIM AND OBJECTIVES

Having identified the gap in knowledge of prestress identification, this research was aimed to develop an innovative non-destructive method to determine the effective prestress force of prestressed concrete box girder bridges utilising their vibration responses.

In order to achieve the above aim, the following objectives were accomplished.

1. Carry out a comprehensive literature review to explore current knowledge on the effect of prestress force on the vibration of structures and study methods of prestress force identification which were developed in previous studies. Further study the applicability of these methods for box girder bridges.

2. Study the effect of prestress force on vibration responses and feasibility of using these vibration responses in an inverse calculation to identify the prestress force of box girder bridges through comprehensive finite element study.
3. Develop a new identification method to quantify the prestress force through inverse calculation and validate through finite element analysis

4. Develop a scale downed version of a prestressed concrete (PSC) box girder bridge and perform vibration tests in laboratory

5. Validate the prestress force identification method (developed in Objective 3) against the experimental results

1.4 SIGNIFICANCE AND SCOPE OF RESEARCH

Bridges form one of the main and essential components in transportation infrastructure systems. They are relatively expensive to construct compared to other components and therefore designed for a longer design life. The performance of these structures reduces with time due to natural, environmental and various other causes such as accidents. However, the demand for road infrastructures is on the rapid rise with heavier vehicles coming onto road day by day.

Among more than 50000 bridges in the Australian road network, some bridges of more than 100 years old are still in service (Pritchard et al., 2014). These old bridges were designed to older design practices and design loads which are significantly different from current standards. Vehicle axle loads on Australian bridges have been increasing at a rate of 10% per decade (Heywood & Ellis, 1998) resulting in much higher traffic loads on these old bridges compared to their original design values. Reflecting the increasing traffic load, Table 1-1 shows the change in design axle load over the time for Australian bridges. On the other hand, failure of bridges has more severe consequences than the failure of road pavements and therefore frequent monitoring and condition assessment of bridges is vital to ensure their safe operation in increasing demand.

When considering bridges in Australian road network, according to Bureau of Transport and Communications Economics (1997), more than 60% of current in-service bridges are prestressed concrete bridges. A comparison of superstructure materials of current bridges in Australia is shown in Figure 1-3.
Table 1-1 Bridge design load variation over time

<table>
<thead>
<tr>
<th>Period</th>
<th>Standard</th>
<th>Design Traffic Load (t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2004-Present</td>
<td>SM1600</td>
<td>160</td>
</tr>
<tr>
<td>1976-2004</td>
<td>T44</td>
<td>44</td>
</tr>
<tr>
<td>1954-1976</td>
<td>H20</td>
<td>33</td>
</tr>
<tr>
<td>1922-1954</td>
<td></td>
<td>15</td>
</tr>
</tbody>
</table>

Figure 1-3 Superstructure Material Usage in Australian Bridges (Adopted from Bureau of Transport and Communications Economics (1997))
Several recent bridge failures all over the world due to defective prestressing systems are alarming authorities to pay their attention to the safety of these structures. For example, Ynys-y-Gwas Bridge in West Glamorgan (Figure 1-4) collapsed in 1985 with no distress sign before the failure. A subsequent investigation found that the prestressing tendons were severely corroded which reduced the load carrying capacity significantly (Woodward & Williams, 1988). Melle bridge in Belgium (Figure 1-5) is another bridge that was suddenly collapsed due to corroded tendons (Schutter, 2013). Hammersmith Flyover in London that is shown in Figure 1-6 is a hollow prestressed concrete bridge which was closed in 2011 after discovering severe damage to prestressing tendons which had a possibility to cause the bridge to collapse (Wikipedia, 2016).

Figure 1-4 Ynys-y-Gwas Bridge collapse (BBC NEWS 2012)
As discussed above, effective prestress force is a key factor that determines the load carrying capacity of prestressed bridges. Defective prestressing systems can cause a sudden collapse of a prestressed bridge without any prior warning. Moreover, traditional methods of visual inspection for assessing the bridge condition are unable
to capture reduction in prestress force until it causes severe damage or failure. Hence a reliable method to assess prestressed bridges is vital for their safe operation.

The findings of this research will enable to determine the effective prestress force in prestressed concrete box girder bridges by using measured vibration responses with a good accuracy.

Unlike for beams, vibration responses of box girder bridges can vary significantly from one to another due to possible variations in the geometry of these bridges such as the shape of the section, the number of boxes, the location of intermediate diaphragms and support conditions. Due to time constraint and practical limitations in testing, the scope of this research is limited to a simply supported box girder of a uniform cross-section with internal un-bonded prestressing. However, it is believed that the methods and techniques developed in this research will become important bases for further development for other box girder bridges.

1.5 THESIS OUTLINE

This thesis consists of 10 chapters. Chapter 1: presents an introduction to the research with a brief discussion on the background of the research. It further illustrates the aim and objectives of this study, the significance of this research and its scope. Previous studies done on relevant areas are discussed in Chapter 2 which summarise their important findings. Chapter 3 discusses the methods adopted to achieve the objectives of this research. Chapter 4 describes the finite element analysis that was carried out to study the effects of prestress force on the vibration characteristics of prestressed structures. Some important aspects in the vibration of box girder bridges and current methods of vibration analysis are summarised in Chapter 5. Chapter 6 presents a new approach to vibration analysis of box girder bridge deck. The methodology of prestress identification and results of finite element verification are given in Chapter 7. Details of the laboratory test model, its construction steps, test procedures and results are presented in Chapter 8. Finally, Chapter 9 concludes the research with some recommendations for future research.
Chapter 2: Literature Review

This chapter reviews the literature on the vibration of prestressed concrete structures with a particular focus on the prestress force effect issues and studies on effective prestress identification. Section 2.1 discusses the effect of prestress force on the vibration characteristics of beam and plate members from a theoretical point of view and experimental observations of different researchers. Effects of some other factors related to the prestressing system are highlighted in section 2.2. Section 2.3 reviews different methods of evaluating residual stresses and effective prestress force of existing structures. Finally, Section 2.4 summarises the key finding in the literature and highlight the identified gap in knowledge.

2.1 EFFECTS OF PRESTRESSING ON FREE VIBRATION CHARACTERISTICS

Effects of prestressing on the free vibration of structures have been a focus for several studies over the past few decades. A number of numerical studies, laboratory and field tests that were done on this matter have identified certain aspects of the effect on vibration responses and modal parameters of prestressed beams. However, a number of contradictory views of different authors have made the real effect unclear.

2.1.1 Effect on Stiffness and Natural Frequency

In theory, the presence of an axial compressive force reduces the stiffness due to the phenomena called “compression softening” and natural frequency reduces accordingly. According to Euler-Bernoulli beam theory (Leissa & Qatu, 2013) and Kirchhoff’s plate theory (Birman, 2011; Ventsel & Krauthammer, 2001), this effect can be expressed mathematically as shown in equation 2-1 for a simply supported beam and equation 2-2 for a simply supported plate on all four sides.
Figure 2-1 Axially loaded beam and plate

\[ \omega_i^2 = \frac{EI}{M} \left( \frac{i\pi}{L} \right)^4 - \left( \frac{i\pi}{L} \right)^2 \frac{N}{M} \]  \hspace{1cm} 2-1

\[ \omega_{m,n}^2 = \frac{D}{M} (\alpha^2 + \beta^2)^2 - \alpha^2 \frac{N}{M} \]  \hspace{1cm} 2-2

Where,

- \( \omega_i \) - \( i \)th natural frequency of vibration
- \( M \) - Mass of the beam (per unit length) or mass of the plate (per unit area)
- \( N \) - Axial force in the beam (N) or in-plane load in plate (N/m)
- \( D \) - Bending stiffness of the plate
- \( \alpha = \frac{m\pi}{a} \) and \( \beta = \frac{n\pi}{b} \) where \( m, n \) are the mode numbers
- \( a, b \) are the dimensions of the plate and \( L \) is the length of the beam as shown in Figure 2-1.

Even though the theoretical prediction is as above, different trends had observed. Confirming above prediction, some researchers (Abraham et al., 1995; Bokaian, 1988; Law & Lu, 2005; NobleNogal & Pakrashi, 2015; Raju & Rao, 1986; Shin et al., 2016) agree that the natural frequency reduces the prestress force. Contradictory to this, some had observed an opposite trend in natural frequency (Hop, 1991; Jang, et al., 2010; Kim et al., 2010; Lu & Law, 2006; Saiidi et al., 1994; Zhang et al., 2012) while some other researchers argue that there is no effect at all (Deak, 1996; Hamed & Frostig, 2006; Li & Li, 2012; Noble et al., 2014).

In an attempt to explain this contradictory behaviour, Saiidi, et al. (1994) suggest that the increase in natural frequency with the prestress force is due to the effect of prestressing on the microcrack closure which results in an increase in stiffness of the beam. Pursuing this further, some other authors (Deak, 1996;
NobleNogalO'Connor et al., 2015) experimentally verified this behaviour of cracked members. Further, they found that the effect of cracks presence up to a certain prestress level only till which the natural frequency increases with prestress force. After that, the member behaves as an uncracked one as all cracks have been closed by the residual compression. Some other authors (Bažant & Cedolin, 1987; Jain & Goel, 1996; NobleNogal & Pakrashi, 2015) argue that the prestress force due to internal bonded tendons act as an internal force that is phenomenologically different from external forces for which the compression softening theory is not valid. On the other hand, un-bonded tendons transfer prestress force to the concrete at end anchorage only with no other connection between tendons and the concrete resulting it to act as an externally applied force which agrees with the compression softening effect (Breccolotti et al., 2009; Materazzi et al., 2009; Miyamoto et al., 2000).

### 2.1.2 Effects on Mode Shape

Governing differential equation for free vibration of an axially loaded beam with arbitrary boundary conditions can be written as (Leissa & Qatu, 2013),

$$EI \frac{\partial^4 y(x,t)}{\partial x^4} + T \frac{\partial^2 y(x,t)}{\partial x^2} = -\rho A \frac{\partial^2 y(x,t)}{\partial t^2} \tag{2-3}$$

Where $EI$ is the flexural stiffness of the beam, $T$ is the axial force, $\rho$ is the density and $A$ is the cross sectional area. $y(x,t)$ is the dynamic deflection of the beam at time $t$.

By separating variables, $y(x,t)$ can be written as

$$y(x,t) = Y(x) F(t) \tag{2-4}$$

where, $Y(x)$ is the mode shape function.

Regarding $F(t)$, for free vibration,

$$F(t) = B_1 \sin \omega t + B_2 \cos \omega t = B \cos(\omega t + \varphi) \tag{2-5}$$

where, $B_1, B_2$ and $B$ are constants, $\omega$ is the natural frequency and $\varphi$ is the initial phase angle.

Substituting equation 2-4 and equation 2-5 in equation 2-3 and simplifying,
\[
EI \frac{\partial^4 Y(x)}{\partial x^4} + T \frac{\partial^2 Y(x)}{\partial x^2} - \rho A Y(x) \omega^2 = 0
\] 2-6

The general solution to above equation 2-6 is in the form,

\[
Y(x) = c_1 \sinh \alpha x + c_2 \cosh \alpha x + c_3 \sin \beta x + c_4 \cos \beta x
\] 2-7

where,

\(c_1, c_2, c_3, c_4\) are constants that depend on the boundary conditions and

\[
\alpha = \left\{-\left(\frac{T}{2EI}\right) + \left[\left(\frac{T}{2EI}\right)^2 + \left(\frac{\rho A}{EI}\right)\omega^2\right]^{1/2}\right\}^{1/2}
\] 2-8

\[
\beta = \left\{\left(\frac{T}{2EI}\right) + \left[\left(\frac{T}{2EI}\right)^2 + \left(\frac{\rho A}{EI}\right)\omega^2\right]^{1/2}\right\}^{1/2}
\] 2-9

From above equation 2-7 to equation 2-9, it is clear that mode shapes of an axially loaded beam vary with the axial force \(T\). However the relation of mode shape to prestress force is complex.

Similarly, governing differential equation for free vibration of a rectangular plate with the in-plane force in \(x\)-direction only can be written as (Ventsel and Krauthammer, 2001, Birman, 2011, Wang and Wang, 2013),

\[
D \left[ \frac{\partial^4 W_{(x,y,t)}}{\partial x^4} + 2 \frac{\partial^4 W_{(x,y,t)}}{\partial x^2 \partial y^2} + \frac{\partial^4 W_{(x,y,t)}}{\partial y^4} - \frac{N_x}{D} \frac{\partial^2 W_{(x,y,t)}}{\partial x^2} \right] = -M \frac{\partial^2 W_{(x,y,t)}}{\partial t^2}
\] 2-10

Where,

- \(D = \frac{Eh^3}{12(1-\nu^2)}\) - Bending stiffness of the plate
- \(W_{(x,y,t)}\) - Displacement of plate in \(z\) direction at time \(t\)
- \(E\) - Modulus of elasticity
- \(h\) - Plate thickness
- \(\nu\) - Poisson’s ratio
For a plate with two edges parallel to $y$ direction are simply supported, displacement can be simplified as,

$$w_{x,y,t} = Y_{(y)} \sin \left(\frac{m\pi x}{a}\right) W_{(t)}$$  \hspace{1cm} 2-11

For free vibration,

$$W_{(t)} = \sin(\omega_{m,n} t + \varphi)$$  \hspace{1cm} 2-12

Where, 
$\omega_{m,n}$ is the natural frequency and $\varphi$ is the initial phase angle and $m, n$ are the number of half sine waves in x and y directions of mode shape.

Substituting equation 2-11 and equation 2-12 in equation 2-10 and simplifying,

$$D \frac{\partial^4 Y_{(y)}}{\partial y^4} - 2D \left(\frac{m\pi}{a}\right)^2 \frac{\partial^2 Y_{(y)}}{\partial y^2} + \left[D \left(\frac{m\pi}{a}\right)^4 - N_x \left(\frac{m\pi}{a}\right)^2\right] Y_{(y)} = 0$$

Let $\frac{m\pi}{a} = \alpha$

Then, equation 2-13 can be re-written as,

$$D \frac{\partial^4 Y_{(y)}}{\partial y^4} - 2D\alpha^2 \frac{\partial^2 Y_{(y)}}{\partial y^2} + \left[\alpha^4(D - \frac{N_x}{\alpha^2}) - M\omega_{m,n}^2\right] Y_{(y)} = 0$$  \hspace{1cm} 2-14

$D - \frac{N_x}{\alpha^2} = D'$ is the reduced stiffness due to the presence of in-plane compressive stress.

Solution to the above equation 2-13 is in the form

$$Y = C_1 Y_1 + C_2 Y_2 + C_3 Y_3 + C_4 Y_4$$  \hspace{1cm} 2-15

Where, 
$C_1, C_2, C_3$ and $C_4$ are constants that depend on the boundary and initial conditions.

$Y_1, Y_2, Y_3$ and $Y_4$ are the solution for the linearly dependent solution of the auxiliary equation which is in the form,

$$r^4 - 2\alpha^2 r^2 + K = 0$$  \hspace{1cm} 2-16

Where,
Hence it is clear that the solution for the above auxiliary equation depends on $K$ which is a function of in-plane stress $N_x$. So that the mode shape $Y_{(y)}$ is also depends on in-plane stress $N_x$.

Pursuing this further, Bokaian (1988) studied the effect of axial force on mode shape of beams with different boundary conditions and observed that the first few mode shapes slightly vary with the axial force. It has observed that the influence of axial force presence only for few modes and the effect is greatest on the fundamental mode and rapidly decreases as the mode number increases (Dall’asta & Leoni, 1999; Kerr, 1976).

\[ K = \alpha^4 \frac{D'}{D} - \frac{M}{D} \omega_{m,n}^2 \]

2.2 EFFECT OF OTHER PRESTRESSING SYSTEM FACTORS

2.2.1 Bonded and Un-bonded Strands

As discussed before, prestressing cables can either be bonded or unbonded. Embedded bonded tendons are the most widely used type as it has the added advantage of better protection from corrosion. On the other hand, un-bonded tendons are used in some structures as they can be visually inspected and replaced at a later time. In recent years, un-bonded post-tensioning in prestressed concrete has become increasingly popular as an efficient method with the development in sheathed strands which possess several advantages such as higher flexibility of tendons, corrosion protection, small friction losses and ability to complete without grouting (Aeberhard et al., 1990).

It has observed that these two types of prestressing affect the vibration characteristics of prestressed elements differently (Breccolotti, et al., 2009; Jain & Goel, 1996; Materazzi, et al., 2009). Not only that the ultimate capacity and behaviour are also different for two types of prestressing (Ghallab & Beeby, 2005; Ng & Tan, 2006a, 2006b). Un-bonded nature in between end anchorages causes the prestress force to transfer onto concrete through anchors only causing it to act as an external load to the structure (Breccolotti, et al., 2009; Lou & Xiang, 2006; Materazzi, et al., 2009) which also results in a uniform strain distribution along the tendon (Walsh & Kurama, 2010). As a result, natural frequency reduces due to
compression softening as expected by the theory. In contrast, bonded tendons are in contact with concrete throughout its length, resulting in the prestress force to act as an internal force for which the compression softening effect is not valid.

2.2.2 Tendon Profile and Eccentricity

A numerical study done by Aalami (2000) revealed that the contribution of the tendon to the response depends on both stress level and the profile. Pursuing this further, a study done by Grace and Ross (1996) found that both the prestress level and the shape of bonded tendon profile affect the natural frequency. According to them, parabolic tendons increase the natural frequency and further increases with the prestress level while eccentric straight tendons reduce the natural frequency and further decrease with increasing prestress level.

However, above results do not agree with the theoretical prediction. It is clear that the prestress force in an eccentric tendon creates a bending moment. Further, it is constant along the beam length for straight tendons and for analysis it can be represented by an equal axial force and a bending moment of constant magnitude as shown in Figure 2-2. According to the equation2-1 and equation 2-3, there is no effect of bending moment to vibration characteristics of a beam. Consequently, there is no effect of eccentricity of prestressing force on the natural frequency. Continuing this, Chan and Yung (2000) state that, initial upward displacement due to prestressing does not have to be considered and moment due to prestressing can be neglected in vibration analysis.

![Effect of eccentricity of tendon profile](image)

Figure 2-2 Effect of eccentricity of tendon profile

Explaining the aforementioned contradictory behaviours, Lu and Law (2006) argue that the physical presence of prestressing tendons has a dual effect on the
natural frequency of a beam. The tendon itself increases the flexural rigidity due to the higher stiffness of steel and hence the natural frequency, but the increase in self-weight and compressive axial load reduces the frequency. If the stiffening effect due to the increase in flexural rigidity has a greater effect over others, it results in a net increase in natural frequency. Further, the eccentricity of tendon changes the mass distribution across the section leading to a higher equivalent moment of inertia which also contributes to a higher natural frequency.

2.3 PRESTRESS FORCE EVALUATION OF EXISTING STRUCTURES

Having identified the importance of prestress force for the safety and well performance of prestressed structures, a number of studies have been conducted to evaluate the prestress force of in-service structures. Some of these studies were ended without success (Abraham, et al., 1995) while some other successful studies have developed different approaches to quantify the prestress. These methods can be broadly categorised as destructive, semi-destructive and non-destructive methods.

Other than these methods, some of the new constructions are being instrumented at the time of construction so that the effective prestress force can be measured directly anytime throughout their service life (Shin et al., 2015).

2.3.1 Destructive Methods
Destructive methods of assessment often employ a gradually increasing load till cracking or ultimate failure of the member. Even though their application mostly limited to laboratory tests, several destructive test records (Aparicio et al., 2002; Chen, 2005; Chen & Gu, 2005; Harries, 2009; Lorenc & Kubica, 2006; Osborn, et al., 2012; Takebayashi et al., 1994) show a good estimation of actual capacity and effective prestress force. Osborn, et al. (2012) tested seven prestressed concrete bridge girders that had been in service for 42 years to determine their effective prestress force. They used a cracking moment test in which a slowly increasing point load was applied at the mid-span of the simply supported bridge girder until a clearly visible vertical crack propagates across the bottom flange. Then the beam was unloaded again so that the induced crack closes due to the prestress force. Then a strain gauge was attached across the so formed crack and reloaded the beam until crack reopened. The stress of the bottom most fibres at which the crack reopened was the used to estimate the effective
prestress force. Testing this way can quantify the effective prestress force accurately. However, it cannot be applied to in-service bridges as it requires damaging the bridge girder.

Aparicio, et al. (2002) tested eight externally prestressed beams in bending up to failure by flexure. They monitored the effective prestress during the loading and observed that the prestressing steel stress increases with the deflection of the beam. Further, the ultimate capacities of beams were obtained experimentally to compare with the current method of analysis. It showed a good agreement with the numerical method that has been proposed to estimate the ultimate capacity of externally prestressed beams by Ramos and Aparicio (1996). Lorenc and Kubica (2006) followed the same test procedure to study the effect of tendon eccentricity for ultimate capacity and concluded that the eccentricity has no clear effect on ultimate capacity. Further, they also confirmed the behaviour of externally prestressed beams observed in the former study. After performing Similar tests on four prestressed beams, Chen and Gu (2005) proposed a simplified way to calculate the ultimate capacity of externally prestressed beams when the effective prestress is known. Inversely, these methods can be used to calculate the effective prestress force if the ultimate failure load is known. However, they may not be possible with current in-service bridges.

2.3.2 Semi Destructive Methods

Semi-destructive methods usually employ a small hole drill into concrete or steel rebars. Stress release at the new free edge created by the drilling is used to calculate the residual stress level. The steel stress relief hole technique (Owens, 1988), the centre hole stress relief method (Owens, 1993), concrete core trepanning technique (Kesavan et al., 2005) and few other methods (Abdunur, 1993; Owens, et al., 1994; Rendler & Viness, 1966; Ryall, 1994) are being used as semi-destructive tests.

The steel stress relief hole technique (Owens, 1988) can be used to measure the residual stress in steel reinforcement. It requires the reinforcing bars to be exposed by removing the concrete cover. Then a 1.57mm diameter hole of 1mm deep around which three strain gauges were pre-attached was drilled into the rebar. The stress released due to the drilled hole was used to calculate the residual stress. This method can be applied to rebars of 20mm diameter or larger.
Owens (1993) introduced “The centre hole stress relief method” for residual stress determination in 1993 which can be used for smaller steel bars or prestressing tendons. This method also requires a 1.6mm diameter hole drilled to a depth of 1mm. But it requires only two longitudinal strain gauges which can be located even on a 5mm diameter bar. This method was then extended to concrete for which a 75mm diameter hole of 50mm deep has to be drilled in. However, the presence of micro-cracks can significantly influence the results and hence a large number of strain gauges are required to place around the hole to reduce this effect. This method shows a very good accuracy with a maximum error of ± 14 N/mm² for stresses in steel and ± 0.3 N/mm² for stresses in concrete.

Unlike above methods, the concrete core trepanning technique proposed by Kesavan, et al. (2005) places the strain gauges radially on the intended 5mm diameter core which is to be removed by a diamond core drilling machine. However, the estimation of residual prestress from only one or two cores is not recommended by the authors as this method has the possibility of introducing some errors. Hence, the authors recommended using a statistical approach with a fairly large number of tests with reasonable reliability.

The above discussed semi-destructive methods that have been proposed in previous studies for residual stress analysis require a permanent damage to concrete or steel or to both which reduce their applicability to real structures.

2.3.3 Non-Destructive Methods in Prestress Evaluation

Non-destructive methods in prestress evaluation are getting increasingly popular over other methods with recent advances in vibration-based structural health monitoring techniques. These methods do no damage to the structure; rather they make use of measurements from externally attached sensors. That could include vibration-based or non-vibration based techniques.

2.3.3.1 Vibration-based techniques in prestress evaluation

With the advances in sensor technologies, vibration-based methods have become popular in civil engineering applications. As a result, a number of studies
have been carried out to evaluate prestress force in prestressed concrete structures using vibration-based methods.

Among those studies, some of them were focused on evaluating the prestress loss in PSC beams (Bruggi, et al., 2008; Caro, et al., 2013; Changchun, 2003; Kim, et al., 2003; Kim, et al., 2004; Wang & Zhou). Loss of prestress cause changes to some structural parameters such as stiffness leading to changes in the natural frequency of vibration. These methods utilise the change in vibrational parameters to calculate the loss in prestress. This requires measurement from two stages to calculate the change in those parameters caused by the loss of prestress. However, this is not available for most of existing bridges. On the other hand, these methods give the prestress loss rather than effective prestress force.

Some other vibration-based methods have been proposed by several researchers (Changchun, 2003; Ho et al., 2012; Jang, et al., 2011; Kim, et al., 2003; Kim, et al., 2004; Law et al., 2007; Law, et al., 2008; Lu, et al., 2008; Lu & Law, 2006; Nedin & Vatulyan, 2013; Velez, et al., 2010; Wang, et al., 2008; Wu, et al., 2008; Xu & Sun, 2011) to evaluate the effective prestress force. Those methods use natural frequency or vibration responses such as acceleration and displacement due to ambient or forced vibration in an inverse calculation to estimate the prestress force indirectly.

In order to estimate the effective prestress of existing structures using measured vibration responses a number of methods have been developed. Some researchers used system identification methods (Ho, et al., 2012; Jang, et al., 2013; Kim, et al., 2004) while some others used model updating methods (Bu & Wang, 2012; Li et al., 2013; Wang, et al., 2008). Direct vibration measurement in an inverse calculation to find the prestress force has also been used successfully in some previous studies (Law & Lu, 2005; Lu, et al., 2008; Lu & Law, 2006). The method proposed by Bu and Wang (2012) is a sensitivity based iterative method which uses vibration responses such as the displacement and the acceleration due to passing vehicles. This method identified the prestress force with a relative error of as low as 1.87%. However, it was a theoretical development only with finite element study. Further, they assumed that the measured responses are noise free which is far from real conditions.
Li et al. (2013) used a sensitivity based model updating method to identify the
prestress force using vibration responses due to a moving vehicle. The method has
been tested for noisy measurement. They identified the prestress force with a
maximum error of 4.11% in just 15 iterations with a 10% noise level. However, this
study was limited to numerical simulations. Further, he considered prestress force as
the only parameter to update the model which assumes that all other parameters of
the model perfectly match with the real structure. This assumption is far from reality
where initial model often associated with a number of complexities and different
degrees of parameter uncertainties for real structures (Kodikara et al., 2016).

System identification method proposed by Ho, et al. (2012) used measured modal
parameters to identify the prestress force. In their method, they first used the
measured change in model parameters to estimate the prestress loss and then used a
system identification approach to identify the baseline model that represents the
target structure. This method identified the prestress force with an error of as low as
1.24%. However, it requires vibration responses at two prestress levels which may
not be possible for existing structures.

The method proposed by Xu and Sun (2011) is also a sensitivity-based method which
requires vibration measurements at two different prestress levels. However, it showed an error of as high as 21.136%.

In the model updating method that was proposed by Wang, et al. (2008), several
different parameters were tested to use in model updating for identify the prestress
force and concluded that the change in the natural frequency as the best in prestress
identification. Again, this method also uses change in natural frequency which
requires test data at two different stages.

Above studies highlight that the model updating and system identification methods
require vibration data measured at two different prestress levels to calculate the
effective prestress force. Hence those methods may not be suitable to identify the
prestress force of most existing structures. However, they may be used with
continuous monitoring to calculate the prestress loss. On the other hand, inverse
methods of prestress identification using measured vibration data require vibration
measurements from current structure only. It does not require a baseline model which
is an added advantage. However, it should be also noted that all the above studies
were focussed on prestressed concrete beams. Prestressed concrete box girder
bridges which are an important type of bridges that are widely being used in road network have not been subjected to study on this aspect to develop a reliable method to identify the effective prestress force. This forms a gap in knowledge which this research was aimed to address.

Among vibration based methods, the method proposed by Law and Lu (2005) which used an indirect method was one of the few accurate methods of identifying the prestress force. Previous vibration based methods i.e. model updating methods and system identification methods used a finite element model of the actual structure to simulate the measured response and the best value for the prestress force to produce equivalent response to the measured response was considered as the prestress force in the real structure. On the other hand, the method proposed by Law and Lu (2005) does not require a finite element model. It rather uses direct vibration responses in an inverse calculation to identify the effective prestress force. This method is more convenient as it does not require any baseline model or previous data and utilises data from the current stage only.

In their study, they used a finite element model of a simply supported prestressed beam to test the proposed method. It generated vibration responses (displacement) due to sinusoidal and impulsive excitations which were then used in the inverse calculation. As shown in Figure 2-3, identified prestress force varied about the actual value. However, it gives a good approximation for the prestress force.
The method proposed by Lu and Law (2006) to identify the PF using measured acceleration responses and strain responses of a simply supported beam also have a good identification potential by utilising measured vibration responses at as less as a single measuring point. This method used a sensitivity based model updating technique to approximate the measured responses which have further verified with laboratory testing.

The new method proposed by Law, et al. (2008) for moving load identification and prestress identification using a wavelet-based method also has a good identification accuracy. Furthermore, it has the advantage of making use of any type of measured dynamic response with no assumption on the initial condition of the system. Figure 2-4 shows the identified prestress force and Figure 2-5 shows the identified axle force in the proposed method.
Other than these, another ongoing research on synergic identification of prestress force and moving vehicle force (Xiang et al., 2015) has also shown a good potential to identify the prestress force and vehicle force at the same time using vibration measurements due to moving vehicle.

Last few studies discussed above are the most recent in prestress identification using vibration-based inverse methods that show good identification accuracy. However, all of these methods were focused on prestressed beams and mostly limited to finite element studies only.
2.3.3.2 Non-vibration based non-destructive methods

Besides vibration methods, some other ways of finding stress in structural materials have also been developed which employ high-frequency waves such as ultrasonic waves (Chang & Liu, 2003; Ciolko & Tabatabai, 1999; Rens & Wipf, 1997).

Ultrasonic methods use the important effect of change in wave velocity of ultrasonic signals due to the stress in the material which is called acoustoelastic effect. These methods have been successfully used to quantify the residual stress in steel and aluminium structures and have not tested on prestressed concrete structures (Bray & Tang, 2001; Chang & Liu, 2003; Crecraft, 1967).

A research that is being conducted by Hussin et al. (2015) has shown that wave mechanism of Lamb waves changes with the prestress force and therefore can be used in prestress identification in concrete structures. According to the authors, transverse and longitudinal wave velocities of lamb waves in concrete are sensitive to the stress in concrete as shown in Figure 2-6 which can be used in prestress evaluation.

![Wave velocity vs Prestress Force](image1)

**Figure 2-6**: Changes in wave velocity with prestress force for longitudinal and transverse waves in concrete.
2.4 SUMMARY AND CONCLUDING REMARKS

This chapter briefly discussed the effects of prestress force on the free vibration of prestressed structures from a theoretical point of view and the different experimental observations and their justifications during previous studies on this matter. Further, prestress identification of prestressed concrete structures has been able to draw the attention of many researchers for few decades which produced a number of interesting articles in the literature. This chapter summarised some of these important studies and discussed their advantages and disadvantages.

Effects of prestress force on the vibration of prestressed beams have been subjected to study for a long time. Theoretically, the presence of an axial force reduces the stiffness due to the phenomena named as “compression softening”. As a result, natural frequency reduces. This change in stiffness causes the vibration responses to change with the prestress force. Therefore, it provides the basis to quantify the effective prestress force in vibration-based methods. However, due to different views of previous researchers, the actual effect of prestressing on the vibration is still not very clear and requires further investigation.

Having identified the importance of effective prestress force, studies on prestress identification have been an interesting focus of researchers which resulted in several methods to evaluate effective prestress force. Some of these methods are destructive methods for which the applications usually are limited to laboratory testing. Some other semi-destructive methods have also been proposed which may be applied to some real structures. However, these methods also make some small
permanent damage to the structure which may not be allowed for most real structures.

With the development of sensor technologies and advances in structural health monitoring techniques, researchers were then interested in non-destructive methods of prestress evaluation. As a result, a number of approaches have been developed. Some of these methods utilised high-frequency waves such as Lamb waves while some others were based on vibration measurements. Advantages and disadvantages of a number of so developed vibration based methods were discussed in this chapter. Among different approaches used in vibration-based methods, inverse methods using direct vibration measurements showed better identification accuracy with no baseline model.

As discussed in Chapter 1, prestressed concrete is now being used in almost all types of civil engineering structures due to its super performances compared to conventional reinforced concrete. However, all above studies on prestressed identification have been focussed on beam-like members only. This was identified as a significant gap in knowledge in this literature review. Hence this research was aimed to fill this gap by extending current knowledge in prestress evaluation to some other types of prestressed structures. With the main aim of developing a comprehensive new approach to evaluating the effective prestress force in box girder bridges which are one of the common prestressed structures, current methods of the evaluation were first extended for plate-like structures. It was then used to identify the prestress force in box girder bridges through new approach for dynamic analysis.
Chapter 3: Research Design

This chapter describes the method adopted by this research to achieve the aims and objectives stated in section 1.3 of Chapter 1. Section 3.1 describes the requirement of the study and Section 3.2 discusses the methodology used to achieve the objectives of this study. Section 3.3 outlines the limitations of the study.

3.1 REQUIREMENT OF STUDY

As discussed in Section 1.4 of Chapter 1, a significant number of old bridges which were designed to old design standards are still in use in Australian road network. These bridges are not only old but also experiencing much higher traffic load than their original intended load. Further, the traffic load is still on the rise at a rate of 10% per decade (Heywood & Ellis, 1998).

As discussed in Chapter 1, prestress force has been identified as an important factor that governs the performance of prestressed structures. A number of bridges have failed in the past due to defective prestressing systems as shown in Section 1.4. Moreover, these types of failures are usually sudden collapses with no prior warning. This has emerged a requirement of the condition and capacity assessment of in-service prestressed bridges for their safe operation. However, lack of knowledge in estimating the effective prestress force in an existing prestressed concrete bridge has become a drawback for capacity assessment.

In order to overcome this, a number of studies have been done in the past. However, as discussed in Chapter 2, those methods were limited for prestressed beams. As discussed in Section 1.2 of Chapter 1, no method has been developed to assess the prestress force of prestressed concrete box girder bridges which are another important type of prestressed bridges. This gap in knowledge highlights the requirement of studying towards developing a prestress identification methodology for box girder bridges.
3.2 METHODOLOGY

In order to address the above requirement of study, the aim and objectives of this research were defined as shown in Figure 3-1 and discussed in Section 1.3 of Chapter 1.

![Figure 3-1 Aim and Objectives]

3.2.1 Objective 1

As discussed in Chapter 2, vibration-based method has been identified as the best approach to use in prestress identification due to its non-destructive manner and proven accuracy for beam-like structures. These methods require a vibration measurement that is sensitive to the effective prestress force which can be used in an inverse calculation to calculate the unknown prestress force. In order to identify that, finite element studies have been carried out using ‘ABAQUS’ finite element software. Effects of different prestressing system parameters such as the tendon profile, the eccentricity of tendons and the bonded between tendons and concrete on the vibration were also studied.
3.2.2 Objective 2

In order to assess the effective prestress force, a good numerical model is vital. In this research, the current methods of prestress evaluation which were limited to beam model have been extended to identify the effective prestress in plate-like structures.

Unlike for beams, there is no current general method for vibration analysis of box girder bridges due to a number of possible geometric variations which make the vibrational behaviour of these structures, unique. However, it has been observed by different researchers and further confirmed during the study that the top slabs of box girders show a plate dominant behaviour. This common feature has been considered in this study to develop a more general approach.

A new approach has been proposed in this study to consider the vibration of the top slab of box girder bridges which utilises characteristic orthogonal polynomials to isolate the top slab for vibration analysis. The proposed method can generate mode shapes of the top slab for vibration analysis with a good accuracy which was then used in an inverse calculation for prestress identification.

3.2.3 Objective 3

Objective 3 was aimed to develop the prestress identification process for box girder bridges using finite element simulation results. In order to validate developed method, a scale downed version of a box girder bridge was tested under laboratory conditions as shown in Figure 3-2.

Lab model was tested at different prestress force levels. The effective prestress forces in tendons were measured using installed load cells. Vibration responses due to a periodic excitation were collected at each prestress level to use in the inverse calculation to verify the proposed method.
3.3 LIMITATIONS

Due to limited time and resources, the scope of this study was limited as discussed in Section 1.4 of Chapter 1. Because of this limited scope, the methods proposed were tested for simply supported box girder bridges with uniform cross section and two end diaphragms only. Further, it considered prestressing with unbonded internal prestressing tendons. The method developed in this study considered prestressing in the longitudinal direction only. It is unable to use for structures with prestressing in two or more directions.

3.4 SUMMARY

After a comprehensive literature review, it was found that no successful method has been developed to assess the effective prestress force in prestressed concrete box girder bridges in a non-destructive manner. In order to address this identified gap in knowledge, this research was aimed to develop a new method to identify the
prestress force in prestressed concrete box girder bridges. To achieve this target, 3 main objectives were set as below.

1. Identify effects of prestressing on vibration characteristics
2. Develop numerical model for prestress force identification
3. Laboratory testing for validation

The methodology that was adopted to achieve these objectives has been summarised in this chapter.
Chapter 4: Effects of Prestress Force on Vibration

Previous studies on the effect of prestress force on vibration that was reviewed in Chapter 2 showed some contradictory observations. In order to verify the actual effects, finite element analysis and parametric studies were carried out as a preliminary study of this research. This chapter shows the details and results of this study.

4.1 BACKGROUND

With the advances in sensor technology, use of vibration based methods for structural damage detection and condition assessment are becoming increasingly popular for civil infrastructures. This has been an interesting area among researchers for few decades. Application of vibration based methods for structural health monitoring is getting wider with contribution from researchers all over the world. Civil engineers have been actively working for several decades on extending these techniques to detect the prestress force in prestressed concrete bridges to ensure their safe operation in increasing road demand.

In order to estimate the effective prestress of existing structures using measured vibration responses a number of methods have been adopted which can be broadly categorised as system identification methods, model updating methods, and inverse methods using direct measurements. All these methods require a parameter sensitive to the prestress force.

As discussed in Section 2.3.3.1, several studies have shown that measured vibration responses have a better potential to be used in indirect calculation to identify the effective prestress force (Bu & Wang, 2012; Law & Lu, 2005; Law, et al., 2008; Li, et al., 2013; Lu, et al., 2008; Lu & Law, 2006; Xu & Sun, 2011). Some researchers (Ho, et al., 2012; Jang, et al., 2011; Kim, et al., 2003) argue that the natural frequency is the best parameter to be used in residual stress estimation while some others recommend measured displacement, strain (Jang, et al., 2010; Law &
or acceleration response (Bu & Wang, 2012; Law, et al., 2008) caused by forced vibration.

Several approaches adopted using different types of responses have given different accuracies in final estimation. Even though the identified prestress forces in these methods tend to fluctuate around the actual force, the average value gives a good estimation. Finite element analysis has been carried out as a part of this research to study the effects of prestress force and other physical parameters on the vibration of prestressed structures and sensitivity of different dynamic responses to identify the best parameters for use in prestress identification.

4.2 NUMERICAL SIMULATION

Effect of prestressing on the vibration of structures has been extensively studied over the past few decades. A number of numerical studies, laboratory and field tests were done on this matter have identified certain aspects of the effect on vibration responses and modal parameters of prestressed beams. However, as shown in chapter 2, some contradictory results have made the real effect unclear.

In order to further investigate the effects of prestressing on vibration characteristics of the prestressed beam, finite element analysis was carried out as it is a powerful and versatile method to analyse any type of structure (Sennah & Kennedy, 2002). A 30m long simply supported AASHTO type v girder (AASHTO, 1996) with end blocks at either end which is used in an existing highway bridge in Sri Lanka was selected so that results are more realistic and reflect the effect of prestress force in a practical range of the prestress force. As a member of the construction team, the author of this thesis was actively engaged in the design review process of this beam and possesses all design parameters for this bridge girder. The bridge was designed according to BS 5400 (BSI, 1978) guidelines. Cross-section of the beam is shown in Figure 4-1. Other design parameters were selected as $E_{\text{conc}} = 34 \text{ GPa}$, $\rho_{\text{conc}} = 2500 \text{ kg/m}^3$, $E_{\text{steel}} = 200 \text{ GPa}$, $\rho_{\text{steel}} = 7800 \text{ kg/m}^3$. Effective prestress force after all losses as per the design calculations is 6231 kN. The prestress force is applied by the means of embedded parabolic tendon with a maximum eccentricity of 482 mm at the mid-span as shown in Figure 4-2.
A finite element model of the beam was developed using the commercially available Finite Element (FE) software “ABAQUS”. ABAQUS has been used for modelling and analysis of prestress concrete structures by many researchers (Choun et al.; Hessheimer et al., 2001; Oliva & Okumus, 2011). The concrete beam was modelled using 3D solid elements, each with 8 nodes (C3D8R) and reduced integration while the tendons were modelled as embedded 3D truss elements, each with 2 nodes (T3D2) as used and recommended in previous studies (Figueiras & Póvoas, 1994; Fu et al., 2015; Lou & Xiang, 2006). ABAQUS has the ability to apply the prestress force to the tendon as an initial stress. The tension in the tendon then transfers as compression to concrete through the perfect bond between concrete and embedded truss elements (Dassault Systèmes, 2012).

Analyses were carried out to study the effects of bonded and un-bonded tendons, the eccentricity of tendon and the tendon profile on the natural frequencies of vibration. Prestress forces are selected as 0, 0.25 F, 0.5F, 0.75 F and F where F is the design effective prestress force after all losses. Figure 4-2 shows the parabolic tendon profile. Stress distribution and the deflection of the original beam due to prestressing are shown (not to a uniform scale) in Figure 4-3.

<table>
<thead>
<tr>
<th>B1</th>
<th>1060</th>
</tr>
</thead>
<tbody>
<tr>
<td>B2</td>
<td>711</td>
</tr>
<tr>
<td>B3</td>
<td>205</td>
</tr>
<tr>
<td>B4</td>
<td>75</td>
</tr>
<tr>
<td>B5</td>
<td></td>
</tr>
<tr>
<td>B6</td>
<td></td>
</tr>
<tr>
<td>D1</td>
<td>1600</td>
</tr>
<tr>
<td>D2</td>
<td>130</td>
</tr>
<tr>
<td>D3</td>
<td>75</td>
</tr>
<tr>
<td>D4</td>
<td>100</td>
</tr>
<tr>
<td>D5</td>
<td>225</td>
</tr>
<tr>
<td>D6</td>
<td>200</td>
</tr>
<tr>
<td>( \gamma_1 )</td>
<td>787</td>
</tr>
<tr>
<td>( \gamma_2 )</td>
<td>813</td>
</tr>
</tbody>
</table>

Figure 4-1 Cross section of AASHTO type V Girder (AASHTO, 1996)
After applying prestress to the beam, it shows an upward deflection as expected and shown in Figure 4-3. The top flange of the girder is subjected to tensile stresses while bottom flange experiencing compressive stress due to the eccentric parabolic tendon profile. Stress in the beam due to prestressing was compared with the design stress. Expected maximum compressive stress after prestressing from theoretical manual calculation was 21.25 MPa whereas finite element (FE) model gave a stress of 21.24 MPa. Hence it shows a very good agreement in stress estimation which confirms the accuracy of modelling of prestress force. Natural frequencies of the non-prestressed beam obtained from the FE model and the theoretically expected values according to Euler-Bernoulli beam theory also agrees well as shown in Table 4-1. Application of un-bonded prestressing reduced the natural frequency of vibration as expected in theory as per the equation 2-1.
Table 4-1 Comparison of natural frequencies (expected and FEM)

<table>
<thead>
<tr>
<th>Mode</th>
<th>Without prestress</th>
<th>With prestress (6231kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Calculated (Hz)</td>
<td>FEM (Hz)</td>
</tr>
<tr>
<td>Mode 1</td>
<td>3.691</td>
<td>3.688</td>
</tr>
<tr>
<td>Mode 3</td>
<td>30.811</td>
<td>30.800</td>
</tr>
</tbody>
</table>

4.3 EFFECTS OF BONDED AND UN-BONDED TENDONS

To study the effects of bonded and un-bonded tendons, the prestress force due to un-bonded tendon was simulated as 3D solid tendons anchored to concrete at both ends. Prestress force was applied using “bolt pre-load” option that is available in ABAQUS which applies the force on the concrete through end anchors and results in a uniform tension in between. Bonded tendons were simulated using embedded 3D truss elements, each with 2 nodes as described in Section 4.2.

Variations of first 3 natural frequencies are shown in Table 4-2. Results show that the natural frequency of vibration marginally increases with prestress force for bonded tendons and it decreases in the case of the un-bonded tendon. As discussed in Chapter 2, similar effects of prestressing on natural frequencies of vibration have been observed in previous studies as well. In an attempt to explain these observations, it has been suggested that the compression softening effect is present in un-bonded tendons only while bonded tendons behave differently (Bažant & Cedolin, 1987; Jain & Goel, 1996; NobleNogal & Pakrashi, 2015). Results of the current FE analysis further confirm this behaviour.
Table 4-2 Effect of prestressed force (F), bonded and un-bonded tendons on natural frequency

<table>
<thead>
<tr>
<th>Mode</th>
<th>Natural frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td>Unbonded</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>3.688</td>
</tr>
<tr>
<td>3</td>
<td>30.800</td>
</tr>
<tr>
<td>Bonded</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>3.688</td>
</tr>
<tr>
<td>3</td>
<td>30.800</td>
</tr>
</tbody>
</table>

4.4 EFFECTS OF ECCENTRICITY OF TENDON PROFILE

When the effect of the change in the 2\textsuperscript{nd} moment of the area due to the change of location of the tendon is neglected, there is no clear effect of eccentricity of tendon path on the natural frequency of prestressed beam with un-bonded tendons. However, when the tendons are bonded, natural frequency increases with the increasing eccentricity of parabolic tendon profile as shown in Table 4-3.

As discussed in Section 2.2.2, in theory, the eccentricity of tendon profile does not affect the natural frequency. The behaviour of un-bonded tendons agrees well with this prediction whereas bonded tendons show a different effect. However, a similar effect as observed in the current study has been observed in a study done by Grace and Ross (1996) in which the natural frequency increased with the eccentricity of bonded parabolic tendons. This result further confirms the differential behaviour of un-bonded tendons and bonded tendons. Only the un-bonded prestressing agrees with the current theoretical explanations while the other shows a clear deviation.
Table 4-3 Effect of eccentricity of tendon profile on natural frequency

<table>
<thead>
<tr>
<th>Mode</th>
<th>Natural frequency (Hz)</th>
<th>e = 0</th>
<th>e = 150mm</th>
<th>e = 285mm</th>
<th>e = 415mm</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>e = 0</td>
<td>e = 150mm</td>
<td>e = 285mm</td>
<td>e = 415mm</td>
</tr>
<tr>
<td>Unbonded</td>
<td></td>
<td>1</td>
<td>3.544</td>
<td>3.545</td>
<td>3.545</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>30.668</td>
<td>30.671</td>
<td>30.671</td>
</tr>
<tr>
<td>Bonded</td>
<td></td>
<td>1</td>
<td>3.693</td>
<td>3.699</td>
<td>3.716</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>30.820</td>
<td>30.864</td>
<td>30.980</td>
</tr>
</tbody>
</table>

4.5 SENSITIVITY OF VIBRATION RESPONSES TO THE PRESTRESS FORCE

A sensitivity study was carried out using developed FE model to further assess the prestressed force effect on vibration responses. The beam was excited with a periodic load of $F(t) = 8000[1 + 0.1 \sin(4\pi t) + 0.05 \sin(15\pi t)]$ N at 12m from left support. Responses were simulated as being recorded at 6m from left support. The sensitivity of displacement (Dis), velocity (V), and acceleration (Acc) are shown in Figure 4-4 to Figure 4-6.

As shown in Figure 4-4 to Figure 4-6, all three vibration responses are sensitive to variation in prestressed force. Hence they have the potential to be used in prestress identification.
Figure 4-4 Sensitivity of displacement with respect to the prestress force

Figure 4-5 Sensitivity of velocity with respect to the prestress force
4.6 PRESTRESS FORCE EFFECT ON BOX GIRDER BRIDGES

As the current study focuses on simply supported box girder bridges only, a finite element analysis of a simply supported box girder with embedded prestressing strands in two webs was carried out to explore the prestress force effects. Box girder section in Figure 4-7 which was used by Bhivgade (2014) in her analysis was used with maximum total prestress force of $F=20000$ kN. End diaphragms of 0.3m thickness were used at both ends. Stress distribution in the box girder due to prestressing is shown in Figure 4-8.

Figure 4-6 Sensitivity of Acceleration with respect to the prestress force

Figure 4-7 Cross-section of the box girder bridge (dimensions are in meters)
Same modelling techniques as that were used to model the beam in Section 4.2 were used to model the box girder bridge. Stress distribution in the box girder bridge due to prestressing shows a similar pattern as expected with tension in top slab and compression in the bottom slab. The first mode of vibration of this box girder was observed as the first bending mode and shown in Figure 4-9. Assuming it as a simply supported hollow beam, calculated natural frequency for the first bending mode using the equation 4-1 is 4.515 Hz. This value is sufficiently close to 4.499 Hz that obtained from FE analysis.

\[
f = \frac{1}{2\pi} \sqrt{\frac{EI}{M}} \left(\frac{\pi}{L}\right)^2
\]

Where,

- \(f\) – First natural frequency
- \(E\) – Elastic modulus
- \(I\) – Second moment of inertia
- \(M\) – Mass of beam per unit length
- \(L\) - Length of beam
A parametric study that was carried out in the same manner as in the study of I-beam showed a similar pattern of prestress effect. Table 4-4 shows the effect of prestress on the natural frequency of vibration of the box girder with bonded and unbonded tendons. Similar to normal beams, un-bonded prestressing and bonded prestressing behaved differently. Un-bonded prestressing reduced the natural frequency with increasing force magnitude while bonded prestressing did not make a notable difference in natural frequency of the first mode but slightly increased the natural frequency of higher modes.

Table 4-4 Effect of prestressing force (F) on box girder bridges

<table>
<thead>
<tr>
<th>Mode</th>
<th>Natural frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td>Un-bonded</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>4.499</td>
</tr>
<tr>
<td>2</td>
<td>11.149</td>
</tr>
<tr>
<td>3</td>
<td>12.659</td>
</tr>
<tr>
<td>Bonded</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>4.499</td>
</tr>
<tr>
<td>2</td>
<td>11.149</td>
</tr>
</tbody>
</table>
4.7 SUMMARY

A number of studies carried out on prestress members had observed that the prestress force affects their vibration characteristics. However, the real effect was not clear due to different observations by different researchers. A finite element analysis was carried out to analyse the effect of prestress force magnitude and other system parameters on the vibration of prestressed beams and box girders bridges. A real I-beam which is used in a current in-service bridge was used in the analysis to reflect the real condition with a practical range of prestressing.

Parametric studies were carried out to identify the effects of system parameters such as bonded and un-bonded tendons, the eccentricity of the tendon on the natural frequency of vibration. Results confirmed the differential effects of bonded and un-bonded tendons on the vibration characteristics of the prestressed structure. Prestressed structures with un-bonded tendons agree with the theoretical prediction of compression softening effect due to which the natural frequency reduces. In opposition to the behaviour of un-bonded tendons, bonded tendons do not make a clear effect on vibration characteristics. It slightly increases the natural frequencies of some modes while some other remains unchanged. Further, this effect does not agree with the theoretical prediction which confirms the observations of some previous studies as discussed in Section 2.2.1.

A sensitivity study that was carried out revealed that vibration responses such as acceleration, velocity and displacement are sensitive to the prestress force. Hence they have the potential to be used in prestress identification.
Chapter 5: Vibration of Box Girder Bridges

In order to use vibration responses of box girder bridges to quantify the effects of prestressing, an accurate analytical model is vital. However, a large range of possible geometric variations of box girder bridges makes the dynamic behaviour of these structures complex. Section 5.1 and 5.2 of this chapter discuss different types of box girder bridges and summarises some commonly use methods of analysis of box girder bridges and their limitations. Section 5.3 discusses some results of a finite element analysis which confirm the deviation of box girder responses from beam approximation.

5.1 BACKGROUND

Box girder bridges are one of the widely used bridge types all over the world due to their aesthetic and superior performance in torsional resistance. Compared to other types of girder bridges, box girders have a longer span and wider deck with minimum materials due to their cellular cross-section.

As shown in Figure 5-1, box girder bridges can be categorised into several types as a single cell, multicell or multi-spine depending on the number of boxes in the cross-section. The behaviour of these types is highly dependent on their cross-sectional configuration and geometry(Cheung & Megnounit, 1991). As a result, there are several methods of analysis for these types of bridges with limited applicabilities for each method.
Not only the cross-sectional geometry but also the longitudinal and horizontal profile, the presence of and locations of intermediate or end diaphragms and type of material also affect the dynamic behaviour of these types of bridges. Figure 5-2 shows some box girder bridges with these types of geometries.
Chapter 5: Vibration of Box Girder Bridges

Figure 5-2 Geometric variations of box girder bridges (Jiang et al., 2014; wordpress, 2015)
5.2 METHODS OF ANALYSIS OF BOX GIRDER BRIDGES

A number of studies in the past on behaviour of box girder bridges have developed different methods to analyse their behaviour. Due to the high geometric dependency of behaviour, most of these methods have limited applicability and accuracies.

The Canadian Highway Bridge Design Code (CSA., 2000) and American Association of State Highway Transportation Officials (AASHTO, 1994, 1996) have recommended several methods to analyse straight box girder bridges. These methods include; Finite strip method, finite element method, orthotropic plate theory, finite difference technique, grillage analysis and folded plate theory.

**Orthotropic plate theory**

In this method, Box Girder Bridge is idealised as an equivalent plate. The stiffness of diaphragm walls is distributed over the length of the plate. This method is mainly limited to multi-spine Bridges. It has been observed that this method gives accurate results for multi-spine bridges with more than 3 spines (Cheung et al., 1982; Sennah & Kennedy, 2002).

**Grillage – Analogy method**

This method is being used as a simple and approximate method to analyse stiffened plates or cellular structures in which the structure is idealised as a grillage. The main disadvantage of this method is the difficulty in representing torsional effects of the structure (Balendra & Shanmugam, 1985). Canadian Highway Bridge Design Code (CSA., 2000) limits the use of this methods to box girder bridges with more than 2 boxes.

**Folded plate theory**

This method utilised the classical two-way plate bending theory along with the plane stress elasticity theory. In this method, the box is considered as a folded plate which is interconnected along their long edges and simply supported at short edges. This method is more complex and time-consuming than other simplified methods. Use of this method is limited to bridges with closely equivalent line supports at their ends (CSA., 2000; Sennah & Kennedy, 2002).
Finite Element Method

Finite element method is a very powerful and versatile method which can be applied to almost all types of structures (Sennah & Kennedy, 2002) and it has been used for analysis for all types of bridges (Cusens & Pama, 1975). Finite element analysis that models the actual cross-section of structure can help recognise the cross-section distortion and its effect on structural behaviour which cannot be recognised from classical methods of analysis (AASHTO, 2012). Due to a large number of equations used in the calculation, manual calculations may not be possible. However, a number of commercially available Finite Element software have made the analysis much easier.

Finite strip method

Finite strip method is considered as a special form of Finite Element method. It is faster in computation than finite element method but less accurate. In this method box girder is divided into finite strips running from one end to the other which are connected along their edges. Displacements of the strip are approximated by a combination of trigonometric functions and polynomials. This method can produce results with reasonably good accuracy (Cheung, 2013).

As summarised above, most of the current methods of analysing box girder bridges are based on a number of simplifications and assumptions which greatly limit the accuracy of these methods for practical application in vibration analysis. Applications of these methods are also limited to certain types of box girder bridges.

5.3 VIBRATION OF BOX GIRDER BRIDGES

Unlike for beams, vibration behaviour of box girders is greatly influenced by the geometry of the section and provision of other structural components such as intermediate diaphragm or cross-frames. For beam-like structures, bending modes are predominant for their behaviour whereas for box girders other modes such as torsion can be more important. According to Canadian Highway Bridge Design Code (CSA., 2001) and some other authors (Cheung & Megnounit, 1991; Fu, et al., 2015; Hewson, 2003a), torsional wrapping effects are often considerable and should not be ignored for box girders.
Box girders are hollow sections with thin walls compared to their other dimensions. They are formed by four plates to resist in-plane or out of plane loading. Hence top and bottom slabs can be treated as plates (Åkesson, 2007; Elgaaly, 1999; Fu, et al., 2015; Lee & Yhim, 2005). On the other hand idealizing box girder as a beam based on the assumption that the cross-section necessarily maintain its original shape under the action of external loads, cannot definitely specify the variation of vibrational behaviour of box girders across a section (Lee & Yhim, 2005). Moreover, it should be noted that most of the current analysis methods of box girder bridges including orthotropic plate method, folded plate method and finite strip method treat them as a plate-like structure.

In order to further study this behaviour, 3 case studies were done using finite element analysis. These bridges were selected to have different cross-sectional geometries with vertical and inclined webs. Further, different span lengths and widths were selected to study the presence of plate behaviour for longer bridges.

Bridge for Case study 1 was from the study done by Bhivgade (2014). It is a 30m long 8.7 m wide rectangular box girder bridge with a span/width ratio of 3.45 which also has vertical webs. A simplified model of the mid-span of Neville Hewitt Bridge in Rockhampton was used in Case study 2. It is a 71m long span with a total width of 9.12m which gives a span/width ratio of 7.78. Case study 3 was done using a simplified finite element model of Kishwaukee River Bridge (USA). It is a 51.8m long, 12.5 m wide bridge with a span/width ratio of 4.14.

**Case study 1**

A finite element model of a simply supported box girder was developed as described in Section 4.6. Dimensions of the box girder were selected to be similar to the one used by Bhivgade (2014) in her analysis. The box girder bridge is 30m long with two end diaphragms. Cross-section of the bridge is shown in Figure 5-3.
Chapter 5: Vibration of Box Girder Bridges

Figure 5-3 Cross-section of the box girder bridge (dimensions are in meters)

Acceleration and displacement responses which were recorded at A, B and C locations as shown in Figure 5-3 for periodic excitation clearly show the variation of dynamic responses across the section. Simulated acceleration and displacement responses that were recorded at the mid-section of the box girder are shown in Figure 5-4 and Figure 5-5 respectively. This variation cannot be accurately described by a beam model.

Figure 5-4 Time history variations of the accelerations at A, B and C

![Acceleration graph](image-url)
Pursuing this further, approximation of simulated responses on the top slab as a beam gave a significant error as shown in Figure 5-6 which shows the simulated displacement at the middle of top slab of prestressed box girder bridge (in red) due to an external periodic excitation and approximated responses as beam with equivalent sectional, material properties and prestress force of same magnitude (in blue).

This research aims to use vibration responses in an inverse calculation to identify the prestress force. As shown in Figure 5-7, change in vibration responses of box girder bridge due to the effects of prestressing is marginal even for the full prestressed level. Hence, an accurate mathematical model to quantify the effect of effective prestress force on the vibration response is vital for developing a reliable prestress identification method.
Figure 5-6 Approximation of time history variation of box girder displacement as a beam

Figure 5-7 Effect of prestressing on the displacement time history of box girder bridge at full prestress level (PF=25MN)
In order to compare the error in beam approximation for the box girder bridge in prestress evaluation, Figure 5-6 and Figure 5-7 were combined in one graph as shown in Figure 5-8 which clearly shows the significance of deviation of box girder behaviour from the beam theory when compared to the effect of prestressing.

Prestress calculation in inverse methods utilising vibration measurements requires measured displacement and acceleration data at a particular time to use in the corresponding governing differential equation to calculate the prestress force. This calculation repeats for a number of data points corresponding to different measurement times to get a representative average value. Further, changes in vibration responses due to the presence of prestress force are proportional to the effective prestress force but comparatively small. For example, as shown in Figure 5-8 at the time 1.2s, the measured displacement on un-prestressed box girder is \(-2.433 \times 10^{-5}\)m and displacement of prestressed box girder is \(-1.45 \times 10^{-5}\)m which gives a difference of magnitude \(0.88 \times 10^{-5}\)m as the effect of prestressing. However, when the box girder is assumed as a beam with equivalent properties, for a same effective prestress force, the expected displacement is \(-0.531 \times 10^{-5}\)m. This has a difference of \(2.08 \times 10^{-5}\)m from the actual displacement of box girder which considers as the error in beam assumption. This difference of \(2.08 \times 10^{-5}\)m is significant when compared to the actual prestress effect of \(0.88 \times 10^{-5}\)m.

Because of this deviation, use of beam approximation for box girder bridges in prestress evaluation leads to a high error which will be further demonstrated in Section 5.4.
On the other hand, vibration responses measured on the webs of the box girder show a very good agreement with beam approximation as shown in Figure 5-9. However, the Bending stiffness (EI) of the best-approximated beam is different from the calculated value from the geometry of the box girder bridge. Due to this difference in geometric stiffness, a beam model is not effective in prestress evaluation of box girder bridges. However, it may be used to monitor the prestress loss if responses at two or more stages are available to find the effective web stiffness which will be discussed in next chapter.
Furthermore, a modal analysis that was carried out to identify the predominant structural behaviour as shown in Figure 5-10 revealed that the behaviour of the top and bottom slabs are closer to the behaviour of a plate whereas the short and long vertical walls (webs) show a beam dominated behaviour.

Figure 5-10 First few mode shapes of the box girder bridge
This can be further confirmed by comparing above mode shapes with the mode shapes of a plate with approximately similar boundary conditions and the mode shapes of a beam with equal length as shown in Figure 5-11. In this analysis, a plate of the same size as the top slab of above box girder was modelled with two long edges cast into a beam. Remaining two short edges were considered as simply supported. It is clear that the behaviours of the plate and the top slab of box girder are apparently similar. However, they are significantly different from the mode shapes of a beam. In addition, Modal Assurance Criteria (MAC) calculated using the mode shapes of these two structures shows a good consistency of mode shapes as shown in Figure 5-12.

Figure 5-11 Comparison of mode shapes
Case study 2

Neville Hewitt Bridge in Rockhampton, Queensland was modelled using ABAQUS software. It is a 71m long prestressed concrete straight box girder with approximately uniform cross-section and no intermediate diagrams. Cross-section geometry of the bridge is shown in Figure 5-13 and the simplified cross-section of the FE model is shown in Figure 5-14.
First few vibration modes of the box girder bridge (left) and equivalent plate supported on two beams along the web line (Right) are shown in Figure 5-15. Because of the longer length of the bridge, first two modes are bending dominated modes. Yet, cross-sectional deformation of the bridge in these two modes cannot be accurately explained if it is considered as a beam. Besides, plate approximation of top slab behaviour better explains this variation.
Figure 5-15 First few vibration modes of Neville Hewitt Bridge (left) and equivalent modes of plate (right)

**Case study 3**

Shown below in Figure 5-16 is another prestressed concrete box girder bridge in the USA (Kishwaukee River Bridge). The 51.8m long outer span of the Kishwaukee River Bridge was used to further study the above behaviour. A simplified model of the bridge as shown in Figure 5-17 was analysed to see its vibration mode shapes. Mode shapes shown in Figure 5-18 shows a good consistency with the modes of the equivalent plate.

Figure 5-16 Cross-section of Kishwaukee River bridge (USA) (adapted from Nair & Iverson, 1982)
Figure 5-17 Simplified model

Mode 1

Mode 2
This further confirms the plate-like behaviour of top slab of box girder bridges. Besides, the plate-like behaviour of structures is governed by the lower thickness of the structure compared to its width. If the thickness of the structure is in the range of width/80 < Thickness < width/8, Kirchhoff’s plate theory is applicable. Top slabs of prestressed concrete box girder bridges are normally in this range.

Confirming these observations, Elgaaly (1999) also states that “top flange of box girders can be treated as long plates supported along their longitudinal edges and subjected to uniaxial compression”.

5.4 BEAM APPROXIMATION FOR BOX GIRDER BRIDGES IN PFI

The vibrational behaviour of box girder bridges shows a significant deviation from the vibration behaviour of box girder bridges which was already discussed in Section 5.3. In order to assess the accuracy of using beam model for the box girder
bridges in prestress identification, further studies were carried out using the finite element model of the box girder that was used in case study 1.

For an axially loaded beam as shown in Figure 5-19, the governing differential equation for vibration is given by equation 5-1.

\[
M \frac{\partial^2 w_{(x,t)}}{\partial t^2} + C \frac{\partial w_{(x,t)}}{\partial t} + N \frac{\partial^2 w_{(x,t)}}{\partial x^2} + EI \frac{\partial^2 w_{(x,t)}}{\partial x^2} = F(t)
\]

5-1

Where, \( N \) is the axial force due to prestressing, \( w_{(x,t)} \) is the time history of displacement in \( Y \) direction and \( F(t) \) is the external excitation force. By utilizing vibration measurement from the beam, prestress force in the beam can be calculated in an inverse calculation as shown by Law and Lu (2005).

For this purpose, the developed finite element model of the box girder was used to generate vibration responses due to an external periodic excitation force. Displacement and acceleration responses were extracted at 3 different locations of the top slab of the box girder. Prestress identification process was then carried out as proposed by Law and Lu (2005) using these vibration measurements. The identified prestress force as in Figure 5-20 shows a large variation.
Figure 5-20 Identified prestress force assuming box girder as a beam

As discussed in Section 5.3, the deviation of box girder top slab behaviour from the beam behaviour results in a high variation in the identified as in Figure 5-21. In other words, assuming box girder behaviour as equivalent to the beam behaviour in prestress evaluation leads to a large error. The average identified force in this way in the above example is $4.26 \times 10^7$N. With compared to the actual force of $2.0 \times 10^7$N, it has an error of 113%. Hence prestress identification methods based on beam theory are not accurate enough for box girder bridges. A new method for this matter has been developed in this study and discussed in details in coming chapters of this thesis.
5.5 SUMMARY AND CONCLUDING REMARKS

The geometry of box girder bridges can vary across a large range due to a number of variable parameters. Unlike for beams, the dynamic behaviour of box girder bridges highly depends on these geometric variations. Different combinations of possible geometric parameters make these bridges unique not only by appearance but also in dynamic behaviour.

When considering beams, common analysis methods such as Euler-Bernoulli beam theory or Timoshenko beam theory are currently being used as general methods which are applicable for most beam structures. However, development of such general methods for box girder bridges is hampered by the above discussed unique behaviour due to possible vast geometric variations. A number of simplified analysis methods that are currently being used to analyse box girder bridges are limited to a very narrow range. Further, a number of simplifications in these methods greatly reduce the accuracy.

A comprehensive FE analysis that has been carried out to study the dynamic behaviour of box girders revealed that the top slab of the box girder behaves like a plate. The Smaller thickness of the top slab compared to its width characterises this plate behaviour. First few mode shapes of box girder bridges were compared with a plate of same dimensions and with approximately similar boundary conditions. Mode shapes of these two structures show very good consistency when compared to the mode shapes of the beam. This plate behaviour of box girder top slab is important and has to be considered for more accurate dynamic analysis. Further, it highlighted that the plate-like behaviour of the top slab of box girder bridges which can be used in developing a more general method of analysis.

On the other hand, the focus of this research which is to use vibration responses in prestress identification requires a more precise and general method of analysis. Considering box girder as a Euler-Bernoulli beam which assumes no cross-sectional deformation does not accurately explain the variation of vibration responses of box girder bridges across a section. Hence, this assumption can give rise to significant errors in prestress identification as the effect of prestressing is also in the same order.
The above error in prestress identification has been studied in details by using finite element analyses. Comparison of measured vibration responses that were measured on the top slab of the box girder bridge shows a significant variation from the beam approximation compared to the change due to the presence of prestress force. This difference results in a higher fluctuation in identified prestress force when the box girder is assumed as a beam. Hence, beam model is not suitable for prestress identification of box girder bridges.
Chapter 6: Dynamic Analysis of Box Girder Bridge Deck - A New Approach

Having identified the plate dominant behaviour of box girder bridges and importance of considering it for accurate dynamic analysis, an improved method has been developed to idealise the top slab of box girder bridges. This chapter discusses the so developed method of analysis in detail. Section 6.1 briefly describes the background for the content of this chapter. Section 6.2 and Section 6.3 discuss the general approach for vibration analysis of plate-like members and current approach use to deal with complex boundary conditions. Section 6.4 describes the proposed approach to idealise the top slab of a box girder bridge for dynamic analysis and Section 6.5 discusses its advantages. Finally, Section 6.6 gives a summary of the content of this chapter.

6.1 BACKGROUND

As discussed in the previous chapter, idealising box girder as a beam does not definitely explain the variation of dynamic responses across a section which is due to the plate dominant behaviour of the top and the bottom slabs. Even though some previous methods such as orthotropic plate method consider the plate-like behaviour of box girders, a number of simplifications and assumptions greatly reduces the accuracy and limits its applicability for specific types of box girders (Sennah & Kennedy, 2002).

In this research, a more general method has been proposed to accurately predict displacement and acceleration responses of box girder deck due to external excitation. The proposed method considers the top slab of the box girder separately for analysis purposes. The effect of rest of the structure is considered as boundary conditions for the separated top slab. Due to the complexity of this boundary effects, orthogonal polynomials have been used to generate mode shapes of the top slab. In principle, this method is applicable to any type of box girder bridge deck. However,
due to time constraints and limited scope of the research, it was tested only for short span simply supported box girders with uniform rectangular cross section and two end diaphragms only.

The proposed method idealises the top slab (or the bottom slab) of the box girder bridge as an orthotropic plate. This method has some similarities of orthotropic plate method that is being used for analysing box girder bridges. However, a review of current methods of analysis of box girder bridges by Sennah and Kennedy (2002) shows that the simplifications that assume in most current analysis methods limit their accuracy and applicability. Unlike to those methods, the new method proposed in this study does not assume such simplifications.

6.2 VIBRATION OF PLATE-LIKE STRUCTURES

Structural members with relatively small thickness compared to its other dimensions (width/80 < Thickness < width/8)(Ventsel & Krauthammer, 2001) are classified as plates and allows the use of Kirchhoff – Love theory of plates for the vibration analysis. For an orthotropic plate member with dimensions and coordinate system as shown in Figure 6-1, governing differential equation according can be written as,

$$D_x \frac{\partial^4 w_{(x,y)}}{\partial x^4} + 2D_{xy} \frac{\partial^4 w_{(x,y)}}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w_{(x,y)}}{\partial y^4} + \rho h \frac{\partial^2 w_{(x,y)}}{\partial t^2} = p_{(x,y,t)}$$  \hspace{1cm} 6-1

Where,

\[ D_x, D_y \] are the bending stiffness of the plate in x and y directions respectively which are given by

$$D_x = \frac{E_x h^3}{12(1-\nu_{xy} \nu_{yx})}$$

$$D_y = \frac{E_y h^3}{12(1-\nu_{xy} \nu_{yx})}$$

$$D_{xy} = D_x \nu_{yx} + \frac{G_{xy} h^3}{6}$$
Use of above equation 6-1 is based on the assumptions of the Kirchhoff’s plate theory which are,

I. Straight lines normal to the mid-surface remain straight after deformation

II. Straight lines normal to the mid-surface remain normal to the mid-surface after deformation

III. The thickness of the plate remains unchanged during deformation.

Solution for the equation 6-1 gives the displacement response of the plate at any X,Y location due to applied external loading $P_{(x,y,t)}$. 

---

**Figure 6-1 Plate element**

- $G_{xy}$: Shear modulus
- $\nu_{xy}$: Poisson’s ratio corresponding to strain in Y direction for a load in X direction
- $w_{(x,y)}$: Displacement of plate in Z direction
- $E$: Modulus of elasticity of plate material
- $\rho$: Mass density of plate material
- $h$: Plate thickness
- $P_{(x,y,t)}$: Externally applied pressure
6.3 BOUNDARY CHARACTERISTIC ORTHOGONAL POLYNOMIALS (BCOP) IN VIBRATION ANALYSIS OF RECTANGULAR PLATES

When considering dynamic of plates, boundary conditions are very important. Vibration characteristics of plates are highly depending on the boundary conditions. However, exact forms of solutions for equation 6-1 are available only for simple boundary conditions such as simply supported boundaries. According to Navier’s method (Ventsel & Krauthammer, 2001) for a closed form of a solution, at least 2 opposite edges have to be simply supported. Any other form of boundary conditions does not possess such form of function for the mode shapes and therefore considered as complex. On the other hand, boundary conditions of the plate in practical applications often deviate from these simple boundary conditions. In order to overcome this, Bhat (1985a) introduced a new method using characteristic orthogonal polynomials (COP) with the Rayleigh-Ritz method which has shown a very good accuracy.

6.3.1 Rayleigh-Ritz Method for Plates

Rayleigh-Ritz method has been using widely to obtain natural frequencies and mode shapes of structures with complex boundary conditions. In this method, deflection function is assumed as a linear combination of assumed mode shapes with arbitrary constants such that it at least satisfies the geometric boundary conditions.

Firstly, Maximum kinetic and potential energies are expressed in terms of arbitrary conditions. Then an expression for natural frequency is obtained by equating maximum kinetic and potential energies. These arbitrary constants can be calculated by considering the stationary condition of natural frequency at natural modes. Although the Rayleigh-Ritz method is useful in a number of cases, it is sometimes hard to get a meaningful shape function. To overcome this problem, orthogonal polynomials are employed as deflection function which is simple and provides better accuracy. These polynomials are constructed using the Gram-Schmidt procedure to satisfy at least geometric boundary conditions.
6.3.2 COP in Rayleigh-Ritz Method

Orthogonal polynomials that are used in Rayleigh-Ritz method have following characteristics.

I. They satisfy at least geometric boundary conditions
II. They are complete
III. Do not inherently violate the natural boundary conditions

A member function is constructed as a simplest polynomial over the domain of the structure to satisfy at least geometric boundary conditions. Higher members are then constructed using Gram-Schmidt procedure and use them in Rayleigh-Ritz method for extraction of natural frequencies and mode shapes. Another advantage of this method is that it can effectively overcome the ill condition problem.

6.4 DYNAMIC ANALYSIS OF BOX GIRDER BRIDGE DECK

Results of finite element analysis which were discussed in preceding chapters highlighted the plate action of box girder bridges in their dynamic behaviour. As discussed in Section 5.3, top and the bottom slabs of box girder bridges can be treated as plates with a proper selection of boundary conditions. These two members are a common feature for any type of box girder bridge. However, the actual boundary conditions of the top slab can vary depending on the geometry of the box girder and often an exact form of solution for the equation 6-1 is not available. Therefore an approximate solution can be obtained by using COP in Rayleigh-Ritz method. In this section, the same simply supported box girder used in the analysis in Chapter 5: has been used to demonstrate the proposed method.

6.4.1 Boundary Conditions of Box Girder Deck

As discussed in Chapter 5, model analysis of the box girder showed the plate behaviour of the top slab while short and long web showed a beam-like behaviour. Hence the boundary condition for the top plate along the long edges can be treated as cast into a simply supported beam. This simplification is also supported by Elgaaly (1999) who explain that “the top flange of box girders can be treated as long plates supported along their longitudinal edges”.
With the directions of the coordinate system defined as shown in Figure 6-2, support conditions along the edge lead to following equations.

![Figure 6-2 Rectangular axis system](image)

- As the plate is cast into the beam, compatibility requires deflection of the plate along the long edge to be same as the displacement of the beam which leads to,

\[
(EI)_b \left[ \frac{\partial^4 w}{\partial x^4} \right] = D \frac{\partial}{\partial y} \left[ \frac{\partial^2 w}{\partial y^2} + (2 - \nu) \frac{\partial^2 w}{\partial x^2} \right] \tag{6-2}
\]

- Compatibility in rotation requires the rotation of plate along the edge to be same as the rotation of the beam. This result,

\[
(GJ)_b \left[ \frac{\partial^3 w}{\partial x^2 \partial y} \right] = D \left[ \frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2} \right] \tag{6-3}
\]
Where,

\[(EI)_b\] - Bending stiffness of equivalent edge beam

\[(GJ)_b\] - Rotational stiffness of the beam

\[G_b = \frac{E}{2(1+v)}\] - is the shear modulus of the beam

\[J\] - Polar moment of inertia of the beam

\[D\] - Bending stiffness of plate

According to Birman (2011), the torsional stiffness of closed profile beams is much higher than that of open profile beams. Low magnitude excitations used in this study caused small deflections and did not make a significant rotation of webs. Hence no rotation of web was assumed for the current study. However, accuracy may be slightly improved by considering the actual rotational stiffness which may have to calculate using measurements from the structure.

To demonstrate the process, the finite element model of the box girder was excited with a periodic loading and the displacement response of the web at the mid-section was recorded. Then the measured response was approximated with the beam theory. The difference between two curves was calculated as a mean squared error which is given by,

\[
\text{MSE} = \frac{1}{n} \sum_{i=1}^{n} (\tilde{y}_i - y_i)^2
\]

\[(EI)_b\] of the equivalent beam was selected so that the MSE is minimum as shown in Figure 6-3. Measured and approximated responses are shown in Figure 6-4. It is observed that the \[(EI)_b\] of the equivalent beam for best approximation show some deviation from the calculated geometric stiffness of the box girder bridge.
In the case of box girder being considered in this analysis, it is simply supported at its ends. Both ends of the box girder have been provided with two fixed end diaphragms to resist torsional distortion. Due to the stiffness of the end diaphragm, assuming no deformation and to be compatible with the movement of the diaphragm, two short edges of the top slab is considered equivalent to simply support. This assumption is also supported by Birman (2011).
6.4.2 Generating COP

After identifying boundary conditions, it is now time to generate characteristic orthogonal polynomials. Rectangular axis system and dimensions of the top plate was selected to be same as shown in Figure 6-2.

When considering vibration of rectangular plates, deflection can be defined as two beam functions with relevant boundary conditions (Bhat, 1985b, Dickinson and Di Blasio, 1986, Wang and Wang, 2013). Those beam functions can be written as a simple trigonometric function for simply supported plates. But when the boundary conditions are complicated, simple functions are not available. In such situations, COPs can be used.

Then the deflection function of the plate can be expressed as,

\[ w(x, y) = \sum_{i=1}^{m} \sum_{j=1}^{n} A_{ij} f_i(x) g_j(y) \]  

6-5

Where, \( x = \zeta / a \), \( y = \eta / b \) and \( \zeta, \eta \) are the coordinates along x and y axes and \( a, b \) are the dimensions of the plate as shown in Figure 6-2. \( f_m(x) \) and \( g_n(y) \) are characteristic shape functions which satisfy the boundary conditions in equation 6-2 and equation 6-3 along the longitudinal ends and along the simply supported short ends displacement and the bending stresses are zero which give rise to equation 6-6 and equation 6-7.

\[ w = 0 \]  

6-6

\[ \frac{\partial^2 w}{\partial x^2} = 0 \]  

6-7

Now the first members of the functions \( f_m(x) \) and \( g_n(y) \) are selected to be the simplest orthogonal polynomials to satisfy the boundary conditions. In general,

\[ f_1(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 \]  

6-8

\[ g_1(y) = b_0 + b_1 y + b_2 y^2 + b_3 y^3 + b_4 y^4 \]  

6-9

The constants \( a_0 - a_4 \) and \( b_0 - b_4 \) have to be determine to satisfy the aforementioned boundary conditions.
Then, the higher members of the orthogonal polynomial in the domain $a,b$ are generated using Gram-Schmidt process as shown below (Bhat, 1985a).

$$f_2(x) = (x - B_2)f_1(x)$$

$$f_i(x) = (x - B_i)f_{i-1}(x) - C_if_{i-2}(x)$$

Where,

$$B_i = \frac{\int_a^b x [f_{i-1}(x)]^2 \phi(x) \, dx}{\int_a^b [f_{i-1}(x)]^2 \phi(x) \, dx}$$

$$C_i = \frac{\int_a^b xf_{i-1}(x)f_{i-2}(x)\phi(x) \, dx}{\int_a^b [f_{i-2}(x)]^2 \phi(x) \, dx}$$

Where, $\phi(x)$ is a weighing function. In the current analysis, plate is assumed to be uniform. Hence the weight function is taken as unity.

### 6.4.3 Obtaining Eigenvalues

Eigenvalues of the idealised plate can then be obtained using above equation (6-5) in Rayleigh-Ritz method.

According to Rayleigh-Ritz method, for the structure to be stable total energy should be a minimum. This total energy is in the form of strain energy and the potential energy. Further, maximum kinetic energy ($T_{\text{max}}$) is equal to the maximum potential energy ($U_{\text{max}}$).

$$T_{\text{max}} = \frac{1}{2} \rho ab \omega^2 \int_0^1 \int_0^1 w^2(x,y) \, dx \, dy$$

$$U_{\text{max}} = \frac{1}{2} D ab \int_0^1 \int_0^1 \left[ \frac{\partial^2 w}{\partial x^2} + \alpha^4 \frac{\partial^2 w}{\partial y^2} + 2\nu a^2 \frac{\partial w}{\partial x} \frac{\partial w}{\partial y} + 2(1 - \nu) a^2 \frac{\partial^2 w}{\partial y \partial x} \right] \, dx \, dy$$
Where,

\[ \alpha = \frac{a}{b} \] is the side ratio

\( h \) is the thickness of the plate

\( \rho \) is the density of the material

\( v \) is the Poisson's ratio

\[ D = \frac{Eh^3}{12(1-v^2)} \] is the flexural rigidity of the plate

Substituting the deflection function in equation 6-5 in terms of orthogonal polynomials and minimising Rayleigh quotient with respect to coefficient \( A_{ij} \) gives the eigenvalue equation. Solution of the eigenvalue equation will give the natural frequencies and mode shapes of the top plate of the box girder.

By following the above-described procedure, first few mode shapes of the top slab of box girder bridge were obtained and are shown in Figure 6-5. Any higher mode shape can be obtained by repeating the procedure for higher \( i, j \) values. The modal assurance criterion for box girder mode shapes and generated mode shapes shows a good agreement as shown in Figure 6-6.
Figure 6-5 Generated mode shapes using COPs

Figure 6-6 Modal Assurance Criterion
Mode shapes of the Kishwaukee Bridge which were discussed in Section 5.3, were also generated in the same way using COP as shown in Figure 6-7. They also show a good consistency.
Figure 6-7 Mode shapes of Kishwaukee Bridge (Left) and Generated mode shapes of top slab (Right)

Figure 6-8 Modal Assurance Criteria
6.5 ADVANTAGES OF PROPOSED METHOD

Orthogonal polynomials have been effectively applied in dealing with complex boundary conditions of structures since Bhat (1985b) introduced this approach in 1985. This research proposed a new approach to use these orthogonal polynomials to idealise the behaviour of top slab of box girder bridges which has not been used before.

This new approach generates mode shapes of top slabs of box girder bridges using characteristic orthogonal polynomials to overcome the complexities due to boundary conditions. Use of orthogonal polynomials in general (Bhat, 1985a; Bhat, 1985b; Chakraverty, 2009; Dickinson & Di Blasio, 1986),

- Provides a simple and efficient approach
- Improves the accuracy of estimation
- Avoid ill condition problem

Dealing with civil structures is always associated with a number of uncertainties. Especially, concrete structures are more prone to have geometric variations and material variations than other materials such as steel. This variation can deviate actual results from an ideal situation significantly. As discussed in Section 6.4.1 the proposed method uses measurements from the real structure to generate mode shapes. Conceptually, use of these measurements from the actual structure can give following advantages. However, more studies are required to confirm these effects.

- Reduces the effect of geometric variations

The proposed new approach uses measured displacement response on the web to idealise the top slab for dynamic analysis. Use of measured response allows reducing the effect of geometric variations as it already contains this effect and will be accounted for in approximating web EI.

- Reduces environmental effects

Similar to the previous effect, measurements from the girder already contain these effects. Therefore they will be automatically accounted by using actual measurements in the approximation process.
• Reduces effect of imperfect support conditions

Similar to above, the effect of support imperfections which already present in measurements will also be accounted for in approximating web stiffness.

Proposed approach idealises the top slab rather than considering full bridge. This can greatly reduce

• Effects of material variation across the section

Box girder bridges are usually large sections. Material properties of concrete can vary from point to point due to various reasons such as construction sequences. Considering top slab only allows avoiding the effect of this variation in the bottom part of the section which will in fact included in the web stiffness approximation process.

• Effects of presence of damage or cracks

Similar to variation in material properties effect of cracks or damages of the bottom section of the box girder will be automatically included in calculations. Hence their effect on top slab analysis will be reduced.

6.6 SUMMARY AND CONCLUDING REMARKS

From the studies that were carried out in Chapter 5: revealed that the dynamic behaviour of box girder is significantly different from the behaviour of beams but show a better constancy with the dynamic characteristics of a plate. This plate-like behaviour of box girder top slab has been considered in this chapter to develop a more general approach to generate mode shape functions of box girder bridge deck for prestress identification which will be discussed in next chapter.

When considering plate-like structures, boundary conditions of the plate are very important for accurate dynamic analysis. However, exact solutions for mode shapes function are available only for a limited number of simple boundary conditions such as simply supported boundaries. Plates with other boundary conditions which do not possess an exact solution are considered as complex. To overcome this complexity, boundary characteristic orthogonal polynomials have
been using successfully in plate vibration analysis since 1985 (Bhat, 1985b) with proven accuracy and some other added advantages. These polynomials can accurately generate mode shape functions for a plate with any boundary condition including simple boundary conditions.

These BCO polynomials have the potential to represent any complex boundary condition. However, they have not been used with box girder bridge decks analysis before. The proposed method in this chapter utilised these polynomials to generate the mode shape functions of the top slab of box girder bridges with good accuracy. Further, the top slab of a box girder bridge which has been used in the proposed method is a common feature for any type of box girder bridges irrespective of its geometry. Hence it has the potential to be extended as a general approach for all types of box girders which is currently not available. However, the current study was limited to simply supported box girder bridges due to time limitations. Further, for the current study, the proposed method was used to generate mode shape functions only and has not developed as a general analysis method.
Chapter 7: Prestress Force Identification from Measured Structural Responses

Having identified the plate-like behaviour of the top slab of box girder bridges a new approach to consider this behaviour was developed in the previous chapter. It is now time to proceed to the main aim of this research which is to identify the prestress force.

This chapter is aimed to develop prestress identification process for box girder bridges. As discussed in previous chapters, the top slab of the box girder bridges can be separated for analysis purposes as a plate by properly treating its boundary conditions. Having done that, basic approach for prestress identification has been developed for a plate-like structure and tested through FE analysis Section 7.2 discusses important steps of this procedure and Section 7.3 shows the verification through FE analysis. Then, Section 7.4 extends and tests this method for box girder bridges. Effects of some other parameters on identification process are discussed in Section 7.5 and finally, Section 7.6 summarises the content of this chapter.

7.1 BACKGROUND

Prestressed concrete is being extensively used as an effective material for different structural elements of almost all types of concrete structures. Plate-like members are one such common type of structural form which is used in many structures, including floor slabs of buildings and bridge decks etc. as shown in Figure 7-1. Chapter 5: of this thesis highlighted that the top slab of box girder bridges also behaves as a plate-like structure. Then a new approach has been proposed in Chapter 6: to idealise this behaviour.

As discussed in Chapter 4: the effect of prestressing on vibration responses is marginal for the practical range of prestressing and requires an accurate mathematical model for quantification of prestress force. However, current methods
of analysis for box girder bridges are simplified approximate methods which are not accurate enough for these purposes. The new approach that has been proposed in Chapter 6 gives a better basis for accurate dynamic analysis of these structures which consider the top slab of a box girder as a plate. Hence this chapter will develop the prestress identification process for a general plate-like structure and test through FE analysis. It will then be used to identify the prestress force in box girder bridges.

Figure 7-1 Common use of plate-like structural elements

7.2 PRESTRESS IDENTIFICATION OF PLATE-LIKE STRUCTURES

Plate-like members in real applications can be of different types such as membrane-like plates, thin plates or thick plates with different boundary conditions. Use of prestressing for concrete structures encourages longer spanning plate-like members with thinner sections leading to higher span to depth ratios with minimum deflections. Lower thickness compared to other dimensions (width/80 < Thickness < width/8) allows the use of Kirchhoff – Love theory of plates for vibration analysis of these members rather than the Mindlin plate theory which is for thicker plates with a thickness of more than 1/8 times its width (Ventsel & Krauthammer, 2001).

Consider a general plate element as shown in Figure 7-2. Governing differential equation for mid-surface deflection of a homogeneous isotropic plate can be written as (Birman, 2011; Timoshenko et al., 1959; Ventsel & Krauthammer, 2001; Wang & Wang, 2013)
\[
D \left[ \frac{\partial^4 w_{(x,y)}}{\partial x^4} + 2 \frac{\partial^4 w_{(x,y)}}{\partial x^2 \partial y^2} + \frac{\partial^4 w_{(x,y)}}{\partial y^4} \right] = P + N_x \frac{\partial^2 w_{(x,y)}}{\partial x^2} + N_y \frac{\partial^2 w_{(x,y)}}{\partial y^2} + 2N_{xy} \frac{\partial^2 w_{(x,y)}}{\partial x \partial y}
\]

7-1

Where,

\[ D = \frac{Eh^3}{12(1-\nu^2)} \] - Bending stiffness of the plate

\[ w_{(x,y)} \] - Displacement of plate in \( z \) direction

\[ E \] - Modulus of elasticity

\[ h \] - Plate thickness

\[ v \] - Poisson’s ratio

\[ P \] - Applied Uniformly Distributed Load (UDL) in \( z \) direction

\[ N_x, N_y, N_{xy} \] - Normal and shear forces

\[ \frac{\partial N_{xy}}{\partial y} \] - Partial derivative of shear force with respect to \( y \)

\[ \frac{\partial N_x}{\partial x} \] - Partial derivative of normal force with respect to \( x \)

Figure 7-2 Plate element with general loading
This can be extended to predict the displacement due to a time-varying force \( P_{(x,y,t)} \) using D’Alembert’s principle as follows.

\[
D \left[ \frac{\partial^4 w_{(x,y,t)}}{\partial x^4} + 2 \frac{\partial^4 w_{(x,y,t)}}{\partial x^2 \partial y^2} + \frac{\partial^4 w_{(x,y,t)}}{\partial y^4} - \frac{N_x}{D} \frac{\partial^2 w_{(x,y,t)}}{\partial x^2} - \frac{N_y}{D} \frac{\partial^2 w_{(x,y,t)}}{\partial y^2} - 2 \frac{N_{xy}}{D} \frac{\partial^2 w_{(x,y,t)}}{\partial x \partial y} \right] = P_{(x,y,t)} - M \frac{\partial^2 w_{(x,y,t)}}{\partial t^2} - C \frac{\partial w_{(x,y,t)}}{\partial t}
\]

7-2

Where,

\( M \) - Mass of plate per unit area

\( t \) – Time

\( C \) – Damping ratio

For the case of prestressed plates with prestressing in one direction only, assuming a plate of the following configuration with dimensions \( a \) and \( b \) as in Figure 7-3, equation 7-2 can be simplified to

\[
D \left[ \frac{\partial^4 w_{(x,y,t)}}{\partial x^4} + 2 \frac{\partial^4 w_{(x,y,t)}}{\partial x^2 \partial y^2} + \frac{\partial^4 w_{(x,y,t)}}{\partial y^4} + \frac{N_x}{D} \frac{\partial^2 w_{(x,y,t)}}{\partial x^2} \right] = P_{(x,y,t)} - M \frac{\partial^2 w_{(x,y,t)}}{\partial t^2} - C \frac{\partial w_{(x,y,t)}}{\partial t}
\]

7-3

Figure 7-3 Plate with pressing in x-direction only

Where \( N_x \) is the in-plane prestress force per unit width of the plate.
Using modal superposition, the solution to the above equation can be written in the form of

$$w_{(x,y,t)} = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \varphi_{m,n(x,y)} W_{m,n(t)}$$  \hspace{1cm} 7-4

Where, $\varphi_{m,n(x,y)}$ is the mode shape function and $W_{m,n(t)}$ is the modal amplitude.

By substituting equation 7-4 in equation 7-3 and simplifying,

$$DW_{m,n(t)} \left[ \frac{\partial^4 \varphi_{m,n(x,y)}}{\partial x^4} + 2 \frac{\partial^4 \varphi_{m,n(x,y)}}{\partial x^2 \partial y^2} + \frac{\partial^4 \varphi_{m,n(x,y)}}{\partial y^4} - \frac{N_x}{D} \frac{\partial^2 \varphi_{m,n(x,y)}}{\partial x^2} \right]$$

$$+ C \varphi_{m,n(x,y)} \dot{W}_{m,n(t)} = P_{(x,y,t)} - M \varphi_{m,n(x,y)} \ddot{W}_{m,n(t)}$$  \hspace{1cm} 7-5

Multiplying equation 7-5 by $\varphi_{m,n(x,y)}$ and integrating over the domain of plate and considering orthogonal property of modes and writing in matrix form,

$$[K_1][W_{(t)}] - N_x[K_2][W_{(t)}] + [C][\dot{W}_{(t)}] = [P] - [M][\ddot{W}_{(t)}]$$  \hspace{1cm} 7-6

Where,

$$K_1 = \int_0^a \int_0^b D \varphi_{m,n(x,y)} \left[ \frac{\partial^4 \varphi_{m,n(x,y)}}{\partial x^4} + 2 \frac{\partial^4 \varphi_{m,n(x,y)}}{\partial x^2 \partial y^2} \right] dx \ dy$$  \hspace{1cm} 7-7

$$K_2 = \int_0^a \int_0^b \varphi_{m,n(x,y)} \left[ \frac{\partial^2 \varphi_{m,n(x,y)}}{\partial x^2} \right] dx \ dy$$  \hspace{1cm} 7-8

$[C]$ is the generalised damping matrix

$[P]$ is the generalised force matrix
\[ N_x[K_2][W(t)] = -[P] + [M][\ddot{W}(t)] + [C][\dot{W}(t)] + [W(t)][K_1] \] 7-9

Let RHS of the equation 7-9 = [A]
And LHS = \( N_x[B] \)

Then using damped least square method for improved accuracy of solution,

\[ N_x = [B]^T[A]( [B]^T[B] + \lambda[I] )^{-1} \]

Where, \( \lambda \) is the non–negative damping coefficient and \( [I] \) is the identity matrix.

The regularized solution for \( N_x \) depends on the regularization parameter \( \lambda \). The convenient way to get the best value for \( \lambda \) is to plot the norm or semi-norm of the solution versus the residual norm \( \| N_x[B] - [A] \| \) which is called the L-curve (Hansen, 1992; Hansen & O’Leary, 1993). An example of a typical L-Curve is shown in Figure 7-4. The value of \( \lambda \) corresponding to the coner of L-curve marked in red in Figure 7-4 gives the best solution.

Figure 7-4 Typical L-Curve - adapted from (Hansen, 1992)
7.3 VERIFICATION AND PARAMETRIC STUDY

A finite element model of a plate similar to that is shown in Figure 7-3 with simply supported edges was used to verify the above-mentioned process of prestress identification using measured displacement and acceleration responses due to periodic excitation.

For simply supported plates, the exact form of deflection function is available in the form,

$$\varphi_{m,n}(x,y) = \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right)$$  \hspace{1cm} 7-10

Then,

$$w_{(x,y,t)} = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) \cdot W_{m,n(t)}$$  \hspace{1cm} 7-11

Where $m$ and $n$ are the number of half sine waves in $x$ and $y$ directions respectively.

Further, external excitation force can be expressed as a double Fourier series as,

$$P_{(x,y,t)} = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} P_{(m,n,t)} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right)$$  \hspace{1cm} 7-12

Where,

$$P_{(m,n,t)} = \frac{4}{ab} \int_{0}^{a} \int_{0}^{b} P_{(x,y,t)} \sin(\alpha x) \sin(\beta y) \ dx \ dy$$  \hspace{1cm} 7-13

By substituting equation 7-12 and equation 7-13 in equation 7-3 and simplifying,

$$M \ddot{W}_{m,n(t)} + (\alpha^2 + \beta^2)^2 \left[ D - \frac{\alpha^2 N_x}{(\alpha^2 + \beta^2)^2} \right] W_{m,n(t)} = P_{m,n,t}$$  \hspace{1cm} 7-14

Where,

$$\alpha = \frac{mn}{a} \ \text{and} \ \beta = \frac{nn}{b}$$
Damping of the plate was not considered in above equation 7-14. Effect of damping was considered as a separate parameter and will be discussed in Section 7.3.6. Comparing equation 7-14 with the general form of the equation of motion, the modal stiffness of the simply supported plate with an in-plane load is

\[
(a^2 + \beta^2)^2 \left[ D - \frac{\alpha^2 N_x}{(a^2 + \beta^2)^2} \right]
\]

\[7-15\]

\[
\frac{\alpha^2 N_x}{(a^2 + \beta^2)^2}
\]

is the reduction in modal bending stiffness due to the effect of in-plane compressive load which is commonly known as compression softening (Materazzi, et al., 2009; Saiidi, et al., 1994).

### 7.3.1 Prestress Force Estimation from Measured Structural Response

Equation 7-4 gives the displacement response \(w_{(x,y,t)}\) at any \((x, y)\) point. By taking first and second derivatives with respect to time, velocity and acceleration of the plate can be expressed as,

\[
\dot{w}_{(x,y,t)} = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \varphi_{m,n}(x,y)W_{m,n}(t)
\]

\[7-16\]

\[
\ddot{w}_{(x,y,t)} = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \varphi_{m,n}(x,y)\ddot{W}_{m,n}(t)
\]

\[7-17\]

In matrix form,

\[
[W]_{p \times 1} = [\varphi]_{p \times q}[W_{(t)}]_{q \times 1}
\]

\[7-18\]

Where \(p\) is the number of measurement locations and \(q\) is the number of modes considered in the calculation.

Generalised modal coordinates matrix can be obtained from equation 7-18 as,

\[
[W_{(t)}] = ([\varphi]^T[\varphi])^{-1}[\varphi]^T[w]
\]

The matrix inversion \(([\varphi]^T[\varphi])^{-1}\) was found as highly ill-conditioned leading to a large error. In order to control the variation of the solution, Tikhonov regularisation is used as in equation 7-19 below.
\[
[W_{(t)}] = \left( ([\varphi]^T[\varphi] + [\Gamma])^{-1}[\varphi]^T[w] \right)
\]

Where the regularisation term \( \Gamma \) is the Tikhonov matrix selected to minimise

\[
\|([\varphi][W_{(t)}] - [w])\|^2 + \|[\Gamma][W_{(t)}]\|^2
\]

Similarly,

\[
[W_{(t)}] = \left( ([\varphi]^T[\varphi] + [\Gamma])^{-1}[\varphi]^T[\dot{w}] \right)
\]

\[
[\ddot{W}_{(t)}] = \left( ([\varphi]^T[\varphi] + [\Gamma])^{-1}[\varphi]^T[\ddot{w}] \right)
\]

Where, \( w, \dot{w} \) and \( \ddot{w} \) are the measured displacement, velocity and acceleration responses respectively.

For an excitation force of \( P_0 f(t) \),

\[
P_{m,n,t} = P_{(m,n)} f(t)
\]

Where,

\[
P_{(m,n)} = \frac{4}{ab} \int_0^a \int_0^b P(x,y) \sin(\alpha x) \sin(\beta y) \, dx \, dy
\]

It can be shown that for a point load at \((\xi, \eta)\) can be written as,

\[
P_{(m,n)} = \frac{4P_0}{ab} \sin(\alpha \xi) \sin(\beta \eta)
\]

Equation 7-14 can be written in matrix form as,

\[
M [\dot{W}_{(t)}] + [D_0 - \alpha^2 N_x] [W_{(t)}] = [P_{m,n,t}]
\]

Where \([D_0] = \text{diag} \left( (\alpha^2 + \beta^2)^2 \right) \). Then,

\[
[\alpha^2 N_x][W_{(t)}] = M [\dot{W}_{(t)}] + [D_0][W_{(t)}] - [P_{m,n,t}]
\]

This is in the form,

\[
[X]N_x = [A]
\]

Where,

\[
[X] = [\alpha^2][W_{(t)}] \text{ and } [A] \text{ is the RHS of equation 7-26.}
\]
Then, $N_x$ can be obtained from damped least square inversion as

$$N_x = ([X]^T[X] + \lambda[I])^{-1}[X]^TA$$

Where, $\lambda$ is the non–negative damping coefficient and $[I]$ is the identity matrix. As discussed in Section 7.2, the value of the regularization parameter $\lambda$ can be obtained by plotting the norm or semi-norm of the solution versus the residual norm (L-Curve).

### 7.3.2 Numerical Simulation

A simply supported plate of 3m x 6m x 0.2m was used to verify the above method and study the effect of the presence of an axial force on vibration. The plate was modelled using shell elements and a uniform in-plane compressive force was applied to simulate the prestress effect. Properties of the plate were selected as $E = 30$ GPa, $\nu = 0.2$ and the mass density of concrete as 2400 kg/m$^3$ which gives a mass per unit area of $M = 480$ kg/m$^2$. A modal analysis was carried out to evaluate the effect of prestressing on the natural frequency of vibration. First six vibration modes and corresponding $m$ and $n$ values are shown in Figure 7-5. Variation of modal frequencies with the axial force is as shown in Table 7-1.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$N_x = 0$ N</td>
</tr>
<tr>
<td></td>
<td>$N_x = 1 \times 10^6$ N</td>
</tr>
<tr>
<td></td>
<td>$N_x = 2 \times 10^6$ N</td>
</tr>
<tr>
<td></td>
<td>$N_x = 4 \times 10^6$ N</td>
</tr>
<tr>
<td>1 1</td>
<td>45.45</td>
</tr>
<tr>
<td>2 1</td>
<td>72.72</td>
</tr>
<tr>
<td>3 1</td>
<td>118.17</td>
</tr>
<tr>
<td>1 2</td>
<td>154.53</td>
</tr>
<tr>
<td>2 2</td>
<td>181.90</td>
</tr>
</tbody>
</table>

Table 7-1 Effect of prestress force on natural frequencies
7.3.3 Prestress Force Identification

The same plate was excited using a sinusoidal periodic excitation force of 5000 \( \sin (15\pi t) \) N at \( P = (2, 1.5) \) to extract the vibration responses for prestress force identification. The coordinates shown in the brackets are in metres. For demonstration purposes, vibration responses were recorded at two sensor locations as
S1 = (1.5, 2.25) and S2 = (4.5, 0.75) as shown in Figure 7-6. Excitation location was selected randomly. There is no specific requirement in selecting this point. However, it is recommended to choose a point toward the middle of the plate to obtain higher-quality response data (i.e. data with a better signal to noise ratio). Measured displacement and acceleration responses at two sensor locations of the prestressed and un-prestressed plate are given in Figure 7-7 and Figure 7-8. Data were recorded at a sampling rate of 1000 Hz.

It is found that a use of six modes leads to better convergence of the double Fourier series which has been employed in equation 7-12 to approximate the excitation force. Hence the first six modes were used in the inverse calculation to estimate the prestress force. Further, the magnitude of the excitation force did not affect the accuracy of results significantly. However, the magnitudes of vibration responses vary with the level of excitation. Consequently, for practical situations, the level of excitation may have to be selected depending on the sensitivity and measurable range of sensors

![Figure 7-6 Sensor layout](image)

In practical applications, vibration data always get polluted with measurement noise and accuracy of the result can be affected. In order to study the effect of noise, a white noise was added to both signals as;

\[ w_{(noisy)} = w_{(calculated)} + noise \]

\[ \ddot{w}_{(noisy)} = \ddot{w}_{(calculated)} + noise \]

The noise was calculated as;
where \( N_r \) is the noise level, \( r \) is a random number drawn from a standard normal distribution with a zero mean and unit standard deviation and \( rms(R) \) is the root mean square value of measured response. Examples of identified forces with and without noise are shown in Figure 7-9. The use of random number \( (r) \) in the noise model generates different noise patterns for a selected noise level at every calculation attempt leading to slightly varied percentage errors. For the purpose of error quantification, calculation has been repeated a number of times (100) and the maximum error was taken as the upper bound of the error due to measurement noise. Average of identified forces as per Figure 7-10 and percentage errors with maximum error, average and standard deviation of percentage errors are shown in Table 7-2.

![Figure 7-7 Measured displacement and acceleration responses at S1](image-url)
Figure 7-8 Measured displacement and acceleration responses at S2

Figure 7-9 Identified prestress force
Table 7-2 Identified average prestress forces and percentage errors (%)

<table>
<thead>
<tr>
<th>Actual prestress force (N/m)</th>
<th>Average prestress force N/m (error)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Without noise</td>
</tr>
<tr>
<td>4.0 x 10^6</td>
<td>4.164 x 10^6 (4.1%)</td>
</tr>
<tr>
<td>2.0 x 10^6</td>
<td>2.191 x 10^6 (9.55%)</td>
</tr>
<tr>
<td>0.4 x 10^6</td>
<td>0.441 x 10^6 (10.25%)</td>
</tr>
</tbody>
</table>

Figure 7-10 Identified prestress forces with a periodic excitation

7.3.4 Optimum Sensor Arrangement

Effect of the number of sensors and the position of these sensors on the accuracy of prediction was studied to optimise the sensor usage. Arbitrarily distributed six sensor locations were selected to extract responses as shown in Figure 7-11. A parametric study carried out reveals that measurements from two measurement locations can predict the prestress force with a good accuracy. Time histories of identified forces with different sensor combinations are shown Figure 7-12. As presented in Figure 7-13 and Table 7-3, use of more measurement may improve the accuracy of identified value but the improvement is fairly marginal in
relation to the number of sensors. On the other hand, the use of only one measurement point should be avoided since this will give comparatively high variation. It is also found that the sensor location can have some effect on the identified results as shown in Figure 7-14. Higher amplitude responses from sensors close to the excitation point such as S1 and S3 in Figure 7-11 results in better identification accuracy than those from other sensor locations such as S2 and S4.

Figure 7-11 Sensor locations

![Figure 7-11 Sensor locations]

Figure 7-12 Effect of number of sensor locations on prestress identification

![Figure 7-12 Effect of number of sensor locations on prestress identification]
Table 7-3 Identified average prestress forces and percentage errors (%) - Effect of number of sensor locations

<table>
<thead>
<tr>
<th>Number of sensor locations</th>
<th>Average prestress force N/m (error)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$1.711 \times 10^6$ (14.45%)</td>
</tr>
<tr>
<td>2</td>
<td>$2.191 \times 10^6$ (9.55%)</td>
</tr>
<tr>
<td>3</td>
<td>$2.173 \times 10^6$ (8.65%)</td>
</tr>
<tr>
<td>4</td>
<td>$2.168 \times 10^6$ (8.40%)</td>
</tr>
<tr>
<td>5</td>
<td>$2.168 \times 10^6$ (8.402%)</td>
</tr>
<tr>
<td>6</td>
<td>$2.168 \times 10^6$ (8.395%)</td>
</tr>
</tbody>
</table>

Figure 7-13 Convergence of error with number of sensors
7.3.5 Effect of Excitation Force

Figure 7-15 shows the identified force for two different excitation magnitudes. Excitation 1 is the same excitation force used in above study and the excitation 2 is 20% lower in magnitude. No significant effect of the magnitude of the excitation force on identified results was observed. However, the magnitude of vibration responses varies with the level of excitation. Consequently, for practical situations, the level of excitation force may have to be selected depending on the measurable range of sensors based on manufacturers specification to capture full response. If the magnitudes of responses are beyond the range of sensors that will cut off the recorded response at its upper and lower limits leaving the mid portion of response only. This can affect the identification accuracy badly as the effect of prestress is more reflected around peaks and minimum points of responses as can be seen in Figure 7-7 and Figure 7-8.
7.3.6 Effect of Damping

Effect of damping ratio on the identification process has been studied to better reflect the applicability to real structures. Prestress identification process for the simply supported plate was repeated considering 2% damping for all modes. Results that are shown in Table 7-4 show a slight increase in error due to damping. However, the effect is very marginal.

Table 7-4 Effect of damping

<table>
<thead>
<tr>
<th>Actual prestress force (N/m)</th>
<th>Average prestress force N/m (error)</th>
<th>Without Damping</th>
<th>With 2% Damping</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.0 x 10^6</td>
<td>4.164 x 10^6 (4.1%)</td>
<td>4.1 x 10^6</td>
<td>4.171 x 10^6 (4.27%)</td>
</tr>
<tr>
<td>2.0 x 10^6</td>
<td>2.191 x 10^6 (9.55%)</td>
<td>2.194 x 10^6 (9.69%)</td>
<td></td>
</tr>
<tr>
<td>0.4 x 10^6</td>
<td>0.441 x 10^6 (10.25%)</td>
<td>0.442 x 10^6 (10.51%)</td>
<td></td>
</tr>
</tbody>
</table>
7.3.7 Identification from Impulsive Excitation

The above method was further verified using an impulsive excitation of magnitude 6000N as shown in Figure 7-16. Use of impulsive excitation is more beneficial than a periodic excitation in terms of practicality. An impulse load can be applied easily by means of drop weight than applying a periodic force which needs special heavy machinery. Effective prestress force can be calculated with a good accuracy using measurements from two locations as shown in Figure 7-17. Neglecting initial estimations with high magnitude due to the initial impact of excitation, the average identified force is $1.91 \times 10^6$ N/m which has an error of as low as 4.5%.

![Figure 7-16 Impulsive excitation](image-url)
7.3.8 Applicability to Different Plate Sizes

The size of these plate structures can vary depending on the application. Long span prestressed floors are a common application. According to the guideline of Cement & Concrete Association Australia (*Design guide for long-span concrete floors*, 1988), prestressed single span flat plates are being commonly used for floors having a span of 6m to 12 m with a most economical range of 6m to 10 m. In order to assess the validity of the proposed method to plates with longer spans in the practical range, two other plates of dimensions 3m x 7.5m and 4m x 12m were studied. Three plates that were studied cover possible short (6m) medium (7.5m) and long (12m) span plates with a span/width ratio of 2, 2.5 and 3 respectively. Time histories of identified prestress forces from periodic excitations are shown in Figure 7-18. Identified forces are of good accuracy with an error in identification as less as 7.41% for the first plate and 7.32% for the second plate. Hence the proposed method does not depend on the size of the plate and applicable to a practical range of prestressed plates.
Prestress identification process has been successfully developed for a plate-like structure and demonstrated for a simply supported plate in above sections. The process demonstrated above for a simply supported plate can be applied to any plate structure with any boundary condition if the modes shape functions are known.

In Chapter 6, a new approach to idealise the top slab of box girder has been developed. Developed method generated mode shape functions as a polynomial of x,y coordinates which can be used in the prestress identification process for the box girder bridges.

Assume that the generated mode shapes are given by \( \varphi_{m,n}(x,y) \). Then, the governing differential equation for vibration of top slab can be written as equation 7-28 below.
\[ DW_{(t)} \left[ \frac{\partial^4 \varphi_{m,n}(x,y)}{\partial x^4} + 2 \frac{\partial^2 \varphi_{m,n}(x,y)}{\partial x^2 \partial y^2} + \frac{\partial^4 \varphi_{m,n}(x,y)}{\partial y^4} - \frac{N_x}{D} \frac{\partial^2 \varphi_{m,n}(x,y)}{\partial x^2} \right] \]

\[ + C \varphi_{m,n}(x,y) \bar{W}_{(t)} = P_{(x,y,t)} - M \varphi_{m,n}(x,y) \ddot{W}_{(t)} \]

7-28

Where,

\( D \) is the bending stiffness of the top slab

\( W_{(t)}, \bar{W}_{(t)}, \bar{\dot{W}}_{(t)} \) are the calculated modal coordinates as per equation 7-19 to equation 7-21.

Multiplying equation 7-28 by \( \varphi_{m,n}(x,y) \) and integrating over the domain of plate and considering orthogonal property of modes,

\[
[K_1][W_{(0)}] - N_x[K_2][W_{(0)}] + [C][\ddot{W}_{(0)}] = [P] - [M][\ddot{W}_{(0)}] \tag{7-29}
\]

Where,

\[
K_1 = \int_0^a \int_0^b \varphi_{m,n}(x,y) \left[ \frac{\partial^4 \varphi_{m,n}(x,y)}{\partial x^4} + 2 \frac{\partial^4 \varphi_{m,n}(x,y)}{\partial x^2 \partial y^2} \right] \ dx \ dy \tag{7-30}
\]

\[
+ \frac{\partial^4 \varphi_{m,n}(x,y)}{\partial y^4} \ dx \ dy
\]

\[
K_2 = \int_0^a \int_0^b \varphi_{m,n}(x,y) \left[ \frac{\partial^2 \varphi_{m,n}(x,y)}{\partial x^2} \right] \ dx \ dy \tag{7-31}
\]

\([C] \) is the generalised damping matrix

\([P] \) is the generalised force matrix

\([M] \) is the generalised mass matrix

Then, the unknown axial force can be calculated as,

\[
N_x[K_2][W_{(0)}] = -[P] + [M][\ddot{W}_{(0)}] + [C][\ddot{W}_{(0)}] + [K_1][W_{(0)}] \tag{7-32}
\]

Let RHS of the equation 7-32 = \([A]\)
Assuming 2% damping for all modes, using a periodic excitation on the same box girder as used in Chapter 6: identified prestress in the top slab of the box girder is shown in Figure 7-19. The average identified prestress force in the top slab is 1.092 MN/m whereas the actual prestress force is 1.075 MN/m. Identified results show a very good accuracy with an error of as small as 1.58%.

Then the effective prestress force can be calculated as,

$$
\text{Effective prestress force} = N_x \times \frac{A}{h}
$$

Where,

- $N_x$ – Identified prestress in top slab per unit width (N/m)
- $h$ – Thickness of top slab
- $A$ – Cross-sectional area of the box girder

---

**Figure 7-19** Identified prestress in the box girder slab
When compared with the prestress identification that was carried out using the beam assumption for box girder bridges which was discussed in Section 5.4, the proposed method shows very good identification accuracy as shown in Figure 7-20. Assuming the box girder bridge as a beam gave rise to an identification error of as high as 113% whereas the error in new method is as small as 1.58%.

![Figure 7-20 Comparison of proposed method and beam assumption for PFI](image)

Similarly, prestress identification was carried out using the developed finite element models of Neville Hewitt Bridge and Kishwaukee River Bridge which were discussed before in section Figure 5-3. The actual structures of these bridges are continuous post-tensioned box girders with embedded bonded tendons and embedded high strength ‘Dywidag’ bars respectively. In this study, they were considered as simply supported bridges post-tensioned with un-bonded tendons to use with the scope of the research. Prestress force of the Neville Hewitt Bridge was kept approximately equal to its design value as per construction drawings. However, No data are available for the prestress force of the Kishwaukee River Bridge. Hence an approximate value has been used. The identified prestress force for Neville Hewitt Bridge is shown in Figure 7-21 while Figure 7-22 shows the results of prestress identification for Kishwaukee River Bridge. Both results show a good identification potential with a reasonable accuracy.
7.5 **EFFECT OF EXCITATION LOCATION AND MAGNITUDE**

Effect of excitation force magnitude on the identification accuracy was tested before for a simply supported plate as discussed in 7.3.5. Further studies were carried out to assess the effect of excitation location and magnitude on prestress identification accuracy for box girder bridges using the developed finite element model.
Excitation locations were selected as shown in Figure 7-23. These locations were selected randomly to represent different areas such as near support, near mid. As discussed in Section 7.3.4, sensors close to excitation leads to better identification accuracy and required a minimum of 3 sensor locations. Hence, for all below studies, measurements from 3 sensor locations arbitrarily selected around the excitation point were used.

Figure 7-24 shows some of the identified prestress forces from measured vibration responses. The result did not show an exact pattern for the effect of excitation location. However, for the same excitation force, the identified results from excitation locations towards the mid of the span shows a better accuracy. When the excitation point is close to the support, identified results show a higher instability as shown in Figure 7-24.
Further, the excitation force magnitude does not make a significant effect on the identification accuracy as shown in Figure 7-25 where Ex2 is a 20% larger force than Ex1.

![Chart showing the effect of excitation force magnitude on Prestress Force Identification](image)

**Figure 7-25** Effect of excitation force magnitude

### 7.6 **SUMMARY AND DISCUSSION**

Plate-like members are one of the main forms of structural elements which are used in vast types of civil structures. Use of prestress concrete has effectively to improve the performance of these types of structure over the conventional reinforced members. As for any other prestressed structure, effective prestress force is the most important parameter that governs the performance of these structures. However, there is no current method to identify the effective prestress force of plate-like structures in non-destructive vibration based methods. This chapter discussed a new approach for the prestress identification of plate-like structures in general and it was then extended to identify the prestress force in box girder bridges using the new method that was developed in former chapters of this thesis.

The proposed method of prestress identification has following advantages.
• Utilises measured vibration responses in a non-destructive manner

• Can be calculated using the measurement from as less as one sensor location. However, the accuracy increases when using more sensors up to 3.

• Independent of the excitation force magnitude

• Robust for measurement noise

• Not-sensitive to prestress force magnitude or the size of the plate.

Further, the studies done on optimal sensor arrangement revealed that the sensors located close to the excitation point results in better identification accuracy due to better signal.

As discussed in previous chapters, the top slab of box girder bridges shows a plate dominant behaviour due to its lower thickness compared to other dimensions. Hence the above-developed procedure of prestress identification can be extended to use with box girder bridges through the new approach of idealising top slab for vibration analysis as proposed in Chapter 6. Extensive studies done on this procedure show a good identification accuracy for different sizes of box girder bridges. Moreover, it was found that the excitation force magnitude has less effect on identification as similar to a simple plate structure. However, for practical applications, the excitation force magnitude will have to be selected to generate vibration responses with a sufficiently large magnitude so that they can be measured in full. Even though the finite element software is capable of capturing responses of very small magnitudes, real sensors have measurable ranges with lower and upper limits. If the generated responses are not in this range, part of the response can be lost and will lead to poor identification accuracy.

It was also found that the excitation locations close to supports of the box girder reduce the stability of the inverse calculation resulting higher variation and less accuracy. This effect can be significant with real structures due to imperfections of support. Hence excitation location has to be selected towards the mid of the bridge. This will also give an added advantage of higher magnitude responses for a lower excitation force.
Chapter 8: Laboratory Testing

After a comprehensive study using finite element models, a method to identify the prestress force was developed successfully. For further studies and validation of the method, scale downed model of a box girder was proposed to test in the laboratory. This chapter briefly presents the design, construction steps of the lab model and procedures, results of the laboratory tests.

8.1 DESIGN OF LAB MODEL

8.1.1 Selection of Size

After a literature review on previous tests that were done on laboratory models of box girder bridges, the cross-section of the lab model was selected as similar to the one used by Madhavi et al. (2006) in their experimental study. However, due to limited resources, for the ease of construction and to suit the requirements of the current study some minor variations were made. The final section of the lab model is shown in Figure 8-1. Length of the girder was selected as 6m due to limited availability of space and for ease of handling.

![Cross-section dimensions of the lab model (dimensions are in millimetres)](image)

Figure 8-1 Cross-section dimensions of the lab model (dimensions are in millimetres)
8.1.2 Reinforcement

For the ease of construction, the box girder was constructed in 3 steps. Hence to ensure monolithic behaviour of the beam and to avoid cracking in handling before prestressing, sufficient longitudinal and shear reinforcements were provided as shown in Figure 8-2. Reinforcement required to resist bursting at prestressing tendon anchorages were checked and provided according to ACI guidelines (ACI Committee 318, 2008).

![Figure 8-2 Reinforcement details](image)

8.1.3 Prestressing Details

In order to study the effect of prestress force on the free vibration characteristics and some other parameters, the lab model has to be prestressed to different prestress levels. Due to limited resources and time, it was decided to make only one model. In order to apply several different prestress forces, un-bonded tendons were considered.

To be more general, tendon profile was selected as parabolic with the maximum possible eccentricity at the middle as shown in Figure 8-3. More detailed drawings are attached in Appendix A.
A 15.2 mm diameter mono strand in an embedded 20mm diameter duct was used in each web. Two steel plates of size 85mm x 150mm x 20mm together with wedge barrels were used at both ends to anchor the tendons as shown in Figure 8-4.
8.2 CONSTRUCTION OF LAB MODEL

After finalising section sizes and reinforcement details, the next step was to plan for the construction. Because of the long hollow section with a small cross-section, pre-planning of construction method was important especially for removing the inner formwork. Considering the ease of construction and removal of formwork, it was decided to build it in 3 stages as shown in Figure 8-5.

![Figure 8-5 Construction stages](image)

As the formwork was supposed to use once only and for ease of handling, plywood formwork with timber supports was used. A drawing of the formwork arrangement is shown in Figure 8-6. Concrete grade was selected as 32MPa concrete with 10mm nominal aggregate size to compact well in narrow vertical sections.

![Figure 8-6 Proposed formwork arrangement](image)
8.2.1 Construction Stage 1

The first step of constructing the lab model was the bottom slab of the box girder. This is an 85mm thick reinforced concrete slab which is connected to the webs along its long edge. In order to avoid cracks and for the box girder to behave as a single unit, sufficient reinforcements were provided across the joint which was continued through the full section. Rebar arrangement is shown in Figure 8-2. Some photos taken during the construction stage 1 are shown below.

Figure 8-7 Reinforcements and formwork for step 1

Figure 8-8 After concreting step 1 and curing
8.2.2 Construction Stage 2

After hardening the concrete of the bottom slab, surface of the hardened concrete along the webs of the box girder was chipped and cleaned to improve the bond between two sections. Then the longitudinal rebars and duct for the prestressing strand were tied onto vertical reinforcements which were continued from the already concreted section. Bellow pictures show some stages during construction step 2.

![Duct for prestressing strands](image1)

Figure 8-9 Installed ducts for prestressing strands

![Formwork for webs](image2)

Figure 8-10 Formwork for webs
8.2.3 Construction Stage 3

Step 3 of construction was to build the top slab of the box girder. Similar to step 2, the surface of the construction joints was made rough to improve the bond.
between two concrete portions (the web and top slab). Formwork for the slab inside the void was erected carefully so that it can be removed easily from the two ends. Some of the photos taken during step 3 are shown below.

![Figure 8-13 Reinforcements and formwork for top slab](image1)

Figure 8-13 Reinforcements and formwork for top slab

![Figure 8-14 Concreting top slab](image2)

Figure 8-14 Concreting top slab
Real box girder bridges are usually provided with diaphragm walls to resist torsional distortions. In addition, provisions of diaphragm walls at supports significantly reduce differential deflections resulting from concentrated loading due to support conditions. Due to construction difficulties and to allow access to the inside of the box girder for placing sensors, it was impossible to build a concrete wall as a diaphragm. Instead, a steel cross frame was provided at the support locations to act as the diaphragm as shown in Figure 8-16. The steel cross-frame was made with 5mm thick 50mm x 50mm steel box sections which were tightly fitted to the box girder model and welded in position.
8.2.4 Prestressing

The main objective of this testing was to validate the proposed method of prestress identification. In order to achieve this, the model was prestressed to different prestress levels and vibration tests were performed at each step.

Prestressing of tendons were carried out at one end using a hydraulic mono jack while the other end of the strand was anchored to the concrete using a wedge barrel and steel plate as shown in Figure 8-17. To compare the accuracy of identification, applied prestress force has to be known. Tensions in strands were measured through cellular load cells that were installed between the live end anchor and the concrete beam as shown in Figure 8-18.

Tensioning of strands was carried out in three steps to get different prestress levels. Prestress forces in strands were measured during tensioning and while testing. Figure 8-19 shows the prestressing of strands and Figure 8-20 shows the measured prestress force during tensioning strand.

Figure 8-17 Dead end anchorage
Figure 8-18 Live end anchorage and load cells

Figure 8-19 Prestressing
8.3 TESTS ON LAB MODEL

8.3.1 Material Testing

In order to test properties of concrete, four test cylinders were cast at each step of concreting. Samples were kept with the test model to have same environmental conditions as the lab model. The samples were then tested for the compressive strength, density and modulus of elasticity as shown in Figure 8-21. Properties of concrete of top slab of the lab model from test result are shown in Table 8-1.
Table 8-1 Properties of concrete of top slab

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Density</td>
<td>2319.58 kg/m³</td>
</tr>
<tr>
<td>Compressive Strength</td>
<td>47.43 MPa</td>
</tr>
<tr>
<td>Modulus of Elasticity</td>
<td>30.6 GPa</td>
</tr>
</tbody>
</table>

8.3.2 Test on Box Girder

The methodology of calculating effective prestress force using vibration responses has been developed and discussed in Chapter 7. The proposed method requires a measurable excitation on the top slab of the box girder.

8.3.2.1 Forced Vibration

In order to test the proposed method, the box girder was excited with a known periodic force. Displacement and acceleration responses were measured at predetermined locations of the top slab. The pictures below show some important components and stages of periodic force vibration and data acquisition.

![Figure 8-22 Sensors on test model](image)

Figure 8-22 Sensors on test model
Figure 8-23 Data acquisition

Figure 8-24 Periodic excitation
Force transfer from the actuator to the test model was through direct contact. During the negative half of the force cycle, if the actuator can be separated from the concrete surface and can produce a strong impact force on the model which may cause damage. In order to avoid this, the model was preloaded with a certain static force and the sinusoidal force was applied on top of the initial preload. The total excitation force was intended to be $[5000 + 3000 \sin(10\pi t)]$ N. However, due to variations from the machine, the actual applied force was $[4979 + 2367 \sin(10\pi t)]$ N. Data was recorded at a sampling frequency of 2000Hz. Sensor locations on the top slab are shown in Figure 8-25. The recorded excitation force is shown in Figure 8-26. Recorded displacement and acceleration responses are shown in Figure 8-27 and Figure 8-28 respectively.

![Figure 8-25 Sensor locations](image)

![Figure 8-26 Excitation force](image)
Above acquired data were oversampled and unavoidably contained high-frequency noise which could be easily removed using a low pass filter with 100 Hz cut-off frequency. Since the excitation frequency was only 5 Hz, this filtering would
not affect the excitation spectrum. Filtered displacement and acceleration responses are shown in Figure 8-29 and Figure 8-30 respectively.

![De-noised displacement](Figure 8-29)

![De-noised acceleration](Figure 8-30)
Modal Analysis

Modal analysis of top slab was carried out at each prestress level to study the effect of prestressing on the natural frequencies and mode shapes. In order to extract frequencies and mode shapes, output-only modal analysis was carried out with random excitation using rubber hammers. Accelerometers were used in every 1m intervals at the mid and on the edge of the top slab for data acquisition. Natural frequencies and mode shapes were then extracted using “ARTeMIS” software. Figure 8-31 shows the accelerometer arrangement for modal analysis of top slab.

![Figure 8-31 Accelerometer arrangement for modal analysis of top slab](image)

8.4 ANALYSIS AND RESULTS

8.4.1 Across the Section Variation of Vibration Responses

Section 5.3 discussed the variation of vibration responses of box girder bridges across a section due to plate behaviour of the slab. This effect was clearly reflected in acceleration responses as shown in Figure 8-32. Variation of displacement responses at two different locations of the same cross-section is shown in Figure 8-33. This difference is due to the cross-sectional deformation of the box girder bridge during vibration which cannot be explained by beam theory as discussed in previous sections of this thesis. However, the difference in two responses is very marginal due to high stiffness of the narrow section and low excitation force magnitude (2367
\( \text{Sin}(10\pi t) \) N). The New approach for analysis that was proposed in Chapter 6 considers this behaviour for more precise analysis.

**Acceleration (g)**

![Graph of variation of measured acceleration across the mid-section](image)

Figure 8-32 Variation of measured acceleration across the mid-section

**Displacement (mm)**

![Graph of variation of measured displacement across the mid-section](image)

Figure 8-33 Variation of measured displacement across the mid-section
8.4.2 Effects of Prestress Force on Natural Frequency

Effects of prestressing on vibration characteristics of prestressed concrete structures have been subjected to studies and discussions for a long period of time. As discussed in Chapter 2: and Chapter 3: theoretical predictions show a reduction in natural frequencies due to the effect of the presence of an axial force which is commonly known as compression softening. Further, Chapter 2: discussed the contradictory behaviour of bonded and un-bonded prestressing which has been observed by previous researchers.

As a part of the laboratory testing program, natural frequency and mode shapes of the box girder bridge model were extracted at 3 different prestress levels as shown in Table 8-2.

<table>
<thead>
<tr>
<th>Tendon 1 (kN)</th>
<th>Tendon 2 (kN)</th>
<th>Total PF (kN)</th>
<th>Direct stress due to prestressing (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PT0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>PT1</td>
<td>142.402</td>
<td>141.607</td>
<td>284.009</td>
</tr>
<tr>
<td>PT2</td>
<td>186.16</td>
<td>192.133</td>
<td>378.293</td>
</tr>
</tbody>
</table>

The output-only modal analysis that was carried out with random excitation captured few mode shapes including first bending and few other coupled modes as shown in Figure 8-34. A description of mode behaviour is given in Table 8-3. Extracted natural frequencies at different prestress levels are shown in Table 8-4 and Figure 8-35 shows them graphically for easy recognition.
Figure 8-34 Mode shapes of top slab of lab model
## Table 8-3 Description of modes

<table>
<thead>
<tr>
<th>Mode</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode 1</td>
<td>1\textsuperscript{st} vertical bending</td>
</tr>
<tr>
<td>Mode 2</td>
<td>Coupled; 1\textsuperscript{st} lateral bending + rigid body motion</td>
</tr>
<tr>
<td>Mode 3</td>
<td>Coupled; lateral sway + twisting around mid-point</td>
</tr>
<tr>
<td>Mode 4</td>
<td>Coupled; anti-end vertical sway + lateral bending</td>
</tr>
<tr>
<td>Mode 5</td>
<td>Coupled; vertical bending (dominant) + 1\textsuperscript{st} lateral bending (minor)</td>
</tr>
<tr>
<td>Mode 6</td>
<td>Coupled; Lateral bending (dominant) + 2\textsuperscript{nd} vertical bending (minor)</td>
</tr>
</tbody>
</table>

## Table 8-4 Natural frequencies (Hz)

<table>
<thead>
<tr>
<th>State\Mode</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>PT0</td>
<td>23.13893</td>
<td>44.5506</td>
<td>57.93582</td>
<td>62.19607</td>
<td>90.81392</td>
<td>95.20988</td>
</tr>
<tr>
<td>PT1</td>
<td>22.97294</td>
<td>43.50206</td>
<td>57.20398</td>
<td>61.43155</td>
<td>84.86443</td>
<td>94.01389</td>
</tr>
<tr>
<td>PT2</td>
<td>22.90249</td>
<td>43.53591</td>
<td>57.23506</td>
<td>61.41358</td>
<td>83.95379</td>
<td>91.71771</td>
</tr>
</tbody>
</table>
Above results show a clear reduction in the natural frequency of 1st bending (mode 1) and other bending dominated modes (mode 5 and mode 6). This confirms the validity of compression softening effect for un-bonded prestressing.

8.4.3 Effect on Vibration Responses

As discussed in section 8.4.1, reduction in the natural frequency with increasing effective prestress force was observed. This reduction is due to the reduced stiffness caused by the presence of the axial force. Consequently, clear changes in displacement and acceleration responses were also observed. Figure 8-36 and Figure 8-37 show the effect of prestress on measured acceleration and displacement responses when the box girder model was prestressed at PT2 (378.293 kN) prestress level.
Figure 8-36 Effect of prestress force on acceleration

Figure 8-37 Effect of prestress force on displacement
8.4.4 Prestress Identification

The process of prestress identification in box girder bridges has been developed and discussed in Chapter 7. The proposed method requires idealising the top slab using boundary characteristic orthogonal polynomials. Boundary conditions of the top slab were considered as simply supported at diaphragm walls and cast into a beam along its longitudinal edges. Similar to the process described in Section 6.4.1, properties of the equivalent beam were calculated by approximating the measured responses with Euler-Bernoulli beam theory to minimise the mean squared error. Approximated displacement in this way is shown in Figure 8-38. Due to the lower width of the section, no significant rotation of plate about the longitudinal edge was observed for the applied excitation force. Hence the rotation of the plate along the web line was assumed as zero.

Displacement (mm)

![Graph showing web displacement](image)

Figure 8-38 Approximation of web displacement considering as a beam

Following the prestress identification process described in Section 7.4, prestress force has then been identified as shown in Figure 8-39 using measured raw data which includes of noise.

Use of Low-frequency excitation (5 Hz) gives an added advantage of easy de-noising which allows using low pass filter to remove most of the high-frequency noises.
Hence, a low pass filter with a cut-off frequency of 100Hz has been used to filter the noise in measured signals. This does not make any significant effect on response spectrum. Identified results using de-noised data are shown in Figure 8-40. Linear approximation (Linear trend lines) for the identified results is also shown.

Figure 8-39 Identified prestress force using raw data

Figure 8-40 Identified prestress forces using de-noised data
### Table 8-5 Accuracy of identification

<table>
<thead>
<tr>
<th></th>
<th>Actual Prestress Force (kN)</th>
<th>Identified force (kN)</th>
<th>Error in identification</th>
<th>Standard deviation (for 10000 data points)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Using Raw Data</td>
<td>284</td>
<td>306.7</td>
<td>7.99%</td>
<td>5.39</td>
</tr>
<tr>
<td></td>
<td>378.3</td>
<td>399.17</td>
<td>5.52%</td>
<td>7.07</td>
</tr>
<tr>
<td>Using De-noised Data</td>
<td>284</td>
<td>302.76</td>
<td>6.61%</td>
<td>3.39</td>
</tr>
<tr>
<td></td>
<td>378.3</td>
<td>397.25</td>
<td>5.01%</td>
<td>4.45</td>
</tr>
</tbody>
</table>

Results of prestress identification that was carried out in the proposed method using the measured test data have very good identification accuracy as shown in Figure 8-39, Figure 8-40 and Table 8-5. During the test, data were recorded at the 2000Hz sampling frequency. Above shown results are for approximately 5 seconds duration. Hence about 10000 data points have been used in standard deviation calculation in Table 8-5. Further, measurement noise does not affect the identification results significantly. The maximum effect of noise increased the error by 1.38% which is very marginal.

### 8.5 GENERALIZED PROCEDURE FOR PRESTRESS IDENTIFICATION

Prestress identification process has been developed and successfully tested through FE analysis and laboratory testing. Application of this procedure has few steps as below.

**Step1- Identify equivalent edge beam properties of top slab**

As discussed in previous sections, proposed method idealised the top slab of box girder bridges as a plate on two edge beams. The bending stiffness of this edge beam is to be determined from field measured data.

For this purpose, box girder has to be excited with a known periodic load and displacement time history of the web due to this periodic load should be measured.
Then, the measured response has to be approximated as a Euler-Bernoulli beam to get the EI for best approximation.

**Step 2 – Generate mode shape function**

Mode shape functions of the top slab should be generated using orthogonal polynomials as shown in Chapter 6. This function will be used to get the modal vectors at sensor locations for inverse calculation

**Step 3 – Data acquisition and inverse calculation**

Displacement and acceleration responses need to be measured due to a known periodic excitation force at 3 or more sensor locations. Excitation locations close to the supports should be avoided. Sensors far from the excitation point can reduce the accuracy and hence should be avoided. The magnitude of the excitation force has to be decided depending on the structure and the measurable range of available sensors. Hence it is highly recommended to carry out a finite element study to decide on the excitation magnitude.

If possible, filtering the measured responses to reduce noise will improve the accuracy of final estimation. Finally, measured data has to be used in an inverse calculation as described in Chapter 7 for prestress identification.

### 8.6 SUMMARY AND DISCUSSION

After developing prestress identification procedure and successfully tested through finite element analysis it was then decided to test through laboratory testing. This chapter discussed the design, construction, test procedures of the lab model and results of laboratory tests.

Due to limited availability of resources, the size of the box girder model was limited to 1m wide and 6m long bridge with sectional dimensions as shown in Figure 8-1. The prestressing system of the lab model was constructed as internal un-bonded tendons with a parabolic tendon profile. Later it was stressed to different prestress levels which were measured through attached load cells. Prestress force levels were selected to apply a prestress of common range for real structures which is 1 to 3 MPa (Aeberhard, et al., 1990; Khan & Williams, 1995). The applied direct stresses on the test model due to the prestress force were 1.58 MPa and 2.1 MPa.
Output-only modal testing and forced vibration testing were carried out to assess the effect of prestressing on the vibration of box girder bridges. Acceleration and displacement responses were recorded at several points on the top slab of the box girder bridge model.

As discussed in Section 5.3 vibration responses measured across a section of the box girder bridge show significant variation from point to point due to the plate-like behaviour of the top slab. This effect was clearly reflected in the measured vibration responses during forced vibration of lab model. Variation of responses was marginal due to the short span, the lower width of the section and the low excitation force, but clear.

Modal test that was carried out to extract natural frequencies of vibration showed a clear effect of un-bonded prestressing on the natural frequency of vibration. Results showed a decreasing trend in natural frequency due to the presence of prestress force which confirms the presence of compression softening effect. This effect was also clearly reflected in measured displacement and acceleration responses during forced vibration.

Lab model was excited with a known periodic force at different prestress force levels. The inverse calculation has been carried out to calculate the prestress force using measured displacement and acceleration responses. Results show a very good accuracy with an identification error of as low as 7.99%. Use of low-frequency excitation enabled to de-noise the collected data using a low-pass filter. This filtering further improved the result. However, the presence of noise did not affect the results significantly. This proves the effectiveness of proposed method with real measurements.
Chapter 9: Conclusions and Future work

9.1 REQUIREMENT OF THE STUDY

Prestressed concrete bridges are one of the commonly used bridge types for many decades due to their better overall performance compared to other types of bridges. As the main factor that governs the performance of these structures, the prestress force and its effects on vibration have been extensively subjected to studying for many years. Having identified the importance, a number of studies have been emerged to quantify the effective prestress force of prestressed concrete beams using their vibration responses. However, the comprehensive literature review that was carried out during this research revealed that those studies were mainly focused on beams. Most of these methods have been limited to theoretical developments which have never been tested for their accuracy. Further, none of these methods have been tested for box girder bridges which show a different vibrational behaviour to that of beams. Therefore, this research aimed to fill this gap in knowledge by developing a novel method to identify the effective prestress force in box girder bridges.

9.2 STUDY APPROACH

In order to achieve the above aim, this research addressed following topics.

- A number of previous studies and tests that were done on the prestress force effect on vibration of prestressed concrete structures have been reviewed during this research. Observations of those studies were further verified through finite element analysis.
- Previous efforts in prestress force identification have been studied to identify their strengths, weaknesses and limitations. Current methods in prestress identification were found to have focussed on the prestressed beam-like
structures only. Hence this research was focused on extending these techniques for prestressed concrete box girder bridges.

- Having identified the strength of vibration-based method in prestress evaluation, the dynamic behaviour of box girder bridges and the effects of prestressing on vibration have been studied using finite element analysis.

- A new approach to idealise the top plate of box girder bridges has been used to treat top slab of the box girder separately which enables to include the plate-like behaviour of the slab more accurately.

- A new approach to calculate the effective prestress force in box girder bridges has been introduced. This method utilises measured vibration responses due to external periodic excitation to estimate the prestress force in an inverse calculation.

- Developed prestress identification method has been further verified through comprehensive lab tests which were done on a 6m long prestressed concrete box girder model.

9.3 KEY FINDINGS AND CONTRIBUTION TO CURRENT KNOWLEDGE

The comprehensive research study that was carried out using finite element software and laboratory tests revealed some important findings. Key findings and contributions to current knowledge by this research can be summarised as follows.

- Effect of prestressing on vibration

  A comprehensive finite element analysis that was carried out to study the effect on prestress force on vibration confirmed that,

  - Internal un-bonded prestressing reduces the stiffness of the structure which results in a reduction in natural frequency. Consequently, this change in stiffness is reflected in the vibration responses.

  - Embedded bonded prestressing tendons act differently to the un-bonded tendons and do not cause a clear change in the stiffness.
Vibration tests that were performed on scaled down lab model of a box girder bridge confirmed the above effects of un-bonded prestressing on vibration.

- Vibration of box girder bridges
  - It was found that the vibration responses measured across a section of the box girder bridge show a clear variation which cannot be explained through beam approximation
  - Above variation was further confirmed through lab testing
  - Results of modal analysis of box girder bridges revealed a plate dominant behaviour of box girder bridges which has to be accounted for in precise dynamic analysis
  - Beam approximation for box girder bridges in prestress force identification resulted in a large error due to the above deviation from beam behaviour

- A new approach to idealise top plate of box girder bridges for more precise dynamic analysis
  - A new approach to generate mode shapes of top slab of box girder bridges has been proposed to idealise the top slab for dynamic analysis
  - Proposed method can accurately generate mode shapes of box girder bridge top slab
  - Proposed method has the potential to be developed as a general approach for bridge deck analysis of box girder bridges

- A new approach to identify prestress force in prestressed plate-like structures
  - A new approach to identify the effective prestress force in prestressed concrete plate-like structures has been proposed
  - The proposed method requires vibration responses measured on the plate due to a known periodic or impulsive excitation
  - Prestress force can be identified using vibration measurement from as less as one sensing location. However, the accuracy improves when using up to 3 sensing locations
Measurement from locations far from the excitation reduces the accuracy of identification due to lower signal strength

- Magnitude of excitation force does affect significantly on identification accuracy
- Proposed method can produce good results even with noisy measurements
- Proposed method is not sensitive to prestress force magnitude or plate size

- Prestress identification of box girder bridges
  - Above method for prestress identification has been successfully extended for box girder bridges through the proposed method of analysing top slab.
  - Proposed method can identify the prestress force in box girder bridges with a good accuracy.
  - PFI in this method requires a known periodical excitation force and vibration measurement from at least 3 locations.
  - The magnitude of excitation does not affect the accuracy of identification.
  - Excitation locations close to the supports of the box girder bridges should be avoided as it can result in poor identification accuracy.

9.4 RECOMMENDATIONS FOR FUTURE STUDIES

It should be noted that due to limited time and resources, the scope of this research was limited to simply supported, straight, single cell box girder bridges with un-bonded prestressing cables. However, the basis of the proposed method is valid for any box girder bridge but it is highly recommended to carry out detailed analysis before applying to box girder bridges with different geometries to confirm the validity. Further, following future work can be proposed to improve the accuracy and applicability of findings of this study.
It has been observed that presence of cracks, environmental effect such as temperature etc. affect the vibration responses of prestressed structures (Noble, et al., 2014; NobleNogalO'Connor, et al., 2015). Properties of concrete in real structures can slightly vary from place to place due to a number of practical reasons such as differential compaction, different concrete supplies etc. Further, cross-sectional sizes of the concrete members can also show some variation from design size. These variations of material properties and uneven sectional dimensions can also have some effect on the vibration responses. These variations are hard to include in the mathematical model directly and hence will give rise to an error. So it may be useful to study the effect of such parameters on identification accuracy which has not been considered during this research. It is recommended to conduct more tests to investigate the effect of these parameters on prestress identification.

Lower thickness compared to other dimensions (width/80 < Thickness < width/8) characterise the plate dominant behaviour of box girder top slab. The method proposed in this research uses this common feature of plate-like slab of all types of box girder bridges which make the proposed method applicable to any box girder bridge. However, due to limited time and resources, it was tested for simply supported, straight, single cell box girders with uniform cross section only. It is recommended to conduct further testing on other types of box girder bridges to validate for all types of box girder bridges.

It has observed that bonded and un-bonded prestressing show different effects on vibration. This effect has to be studied further with more experimental analysis to verify the actual behaviour.

Proposed method in prestress identification in this research is valid for un-bonded prestressing only. However, a number of current bridges have bonded tendons. Hence it would be beneficial to extend these methods for bonded prestressing as well.

The new approach used in this study to idealise top slab of box girder bridges was tested on a straight single cell box girder only. Further studies can be recommended to extend for other types of structures and generate more general approach.
Results of this study identified the prestress force in the box girder bridge as one resultant force with a very good accuracy. However, prestress force applies to structures by means of several tendons. The proposed method is unable to differentiate the force in the individual tendon. It is worth to study further to differentiate the identified prestress force to individual tendons.
Chapter 10: Conclusions and Future works


ACI Committee 318, A. C. I. (2008). *Building Code Requirements for Structural Concrete (ACI 318-08) and Commentary*: American Concrete Institute.


Appendices

Proposed lab model for prestressed concrete box girder

Reinforcement details

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Appendix

Prestressing details

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<th>X (mm)</th>
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<th>500</th>
<th>750</th>
<th>1000</th>
<th>1250</th>
<th>1500</th>
<th>1750</th>
<th>2000</th>
<th>2250</th>
<th>2500</th>
<th>2750</th>
<th>3000</th>
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</thead>
<tbody>
<tr>
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<td>222</td>
<td>209</td>
<td>194</td>
<td>184</td>
<td>173</td>
<td>163</td>
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Proposed lab model for prestressed concrete box girder

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